

# High-Order Conversion From Boolean to Arithmetic Masking

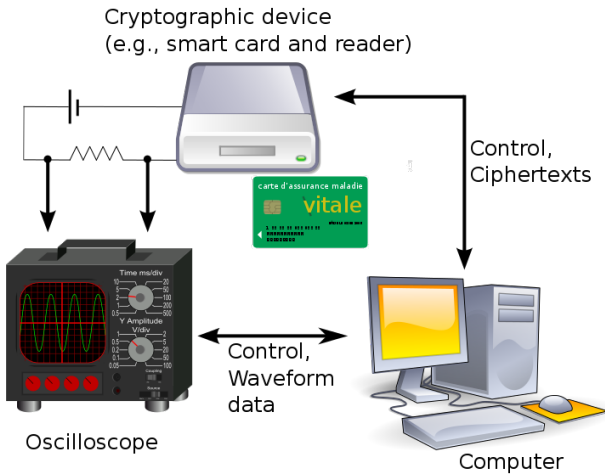
Jean-Sébastien Coron

University of Luxembourg

CHES 2017

Speaker: Srinivas Vivek

# Side-channel Attacks



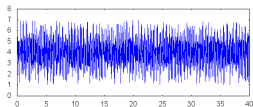
# Differential Power Analysis [KJJ99]

Group by predicted  
SBox output bit

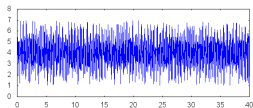
111



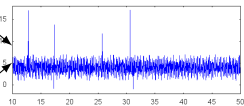
Average trace



000



Differential trace



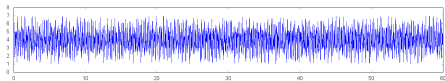
# Masking Countermeasure

- Let  $x$  be some variable in a block-cipher.
- Masking countermeasure: generate a random  $r$ , and manipulate the masked value  $x'$

$$x' = x \oplus r$$

instead of  $x$ .

- $r$  is random  $\Rightarrow x'$  is random  
 $\Rightarrow$  power consumption of  $x'$  is random



$\Rightarrow$  no information about  $x$  is leaked

## Masking Countermeasure

- How do we compute with  $x' = x \oplus r$  instead of  $x$  ?
- Linear operation  $f(x)$  (e.g. MixColumns in AES): easy

$$f(x') = f(x) \oplus f(r)$$

- We compute  $f(x')$  and  $f(r)$  separately.
  - $f(x)$  is now masked with  $f(r)$  instead of  $r$ .
- Non-linear operations (SBOX): randomized table [CJRR99]

# Arithmetic Masking

- Some algorithms use arithmetic operations, for example IDEA, RC6, XTEA, SPECK, SHA-1.
- For these algorithms, we can use arithmetic masking:

$$x = A + r \bmod 2^k$$

where we manipulate  $A$  and  $r$  separately.

- Problem: how do we convert between Boolean and arithmetic masking ?
  - Goubin's algorithm (CHES 01): first-order secure conversion between Boolean and arithmetic masking.

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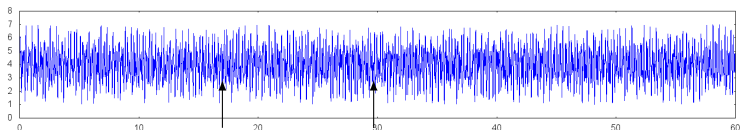
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## Second-order Attack

- Second-order attack:



$E(x')$        $E(r)$

$f(E(x'), E(r))$  correlated with  $x = x' \oplus r$

- Requires more curves but can be practical



## Higher-order masking

- Solution:  $n$  shares instead of 2:

$$x = x_1 \oplus x_2 \oplus \dots \oplus x_n$$

- Any subset of  $n - 1$  shares is uniformly and independently distributed
  - If we probe at most  $n - 1$  shares  $x_i$ , we learn nothing about  $x$
  - $\Rightarrow$  secure against a DPA attack of order  $n - 1$ .

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# Higher-order masking

- High-order Boolean masking:

$$x = x_1 \oplus x_2 \oplus \dots \oplus x_n$$

- High-order arithmetic masking:

$$x = A_1 + A_2 + \dots + A_n \text{ mod } 2^k$$

- Problem: how do we convert between Boolean and arithmetic masking ?
- **This talk:** efficient and provably secure high-order conversion from Boolean to arithmetic masking.

## Prior work and this talk

$n$ : number of shares

$k$ : arithmetic modulo  $2^k$  ( $k = 32$  for HMAC-SHA-1).

|                               | Direction                           | First-order complexity | High-order complexity           |
|-------------------------------|-------------------------------------|------------------------|---------------------------------|
| Goubin's algorithm<br>[Gou01] | B $\rightarrow$ A                   | $\mathcal{O}(1)$       | -                               |
|                               | A $\rightarrow$ B                   | $\mathcal{O}(k)$       | -                               |
| [CGV14]                       | B $\rightarrow$ A                   | -                      | $\mathcal{O}(n^2 \cdot k)$      |
|                               | A $\rightarrow$ B                   | -                      | $\mathcal{O}(n^2 \cdot k)$      |
| [CGTV15]                      | B $\rightarrow$ A                   | -                      | $\mathcal{O}(n^2 \cdot \log k)$ |
|                               | A $\rightarrow$ B                   | $\mathcal{O}(\log k)$  | $\mathcal{O}(n^2 \cdot \log k)$ |
| <b>This talk</b>              | <b>B <math>\rightarrow</math> A</b> | -                      | $\mathcal{O}(2^n)$              |

- **This talk**: complexity of Boolean to arithmetic conversion independent of  $k$ , as in Goubin's first-order algorithm.
- $\mathcal{O}(2^n)$ : exponential complexity, but one order of magnitude faster than prior work for small values of  $n$  !

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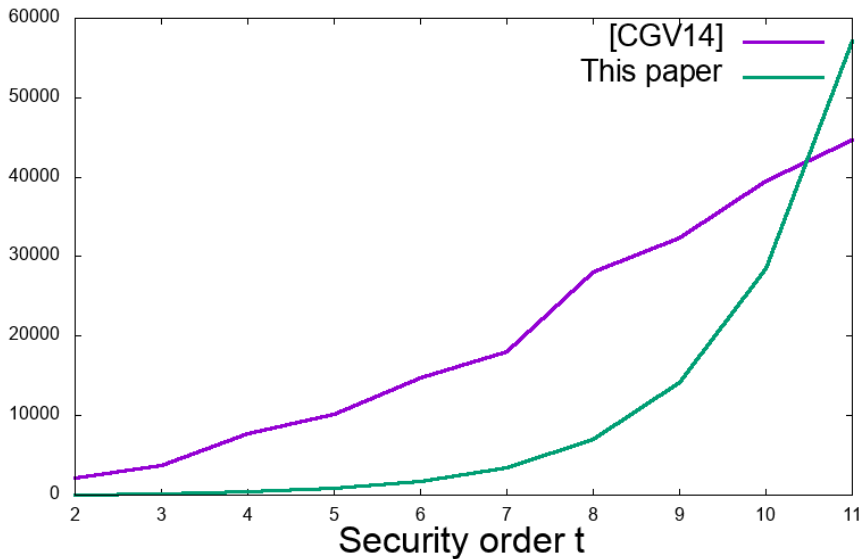
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## Comparison with prior work ( $k = 32$ bits)



# Our contribution

- Our contribution: high-order conversion algorithm from Boolean to arithmetic masking
- with a proof of security in the ISW probing model.
- Our algorithm is a variant of an algorithm published on ePrint by Hutter and Tunstall [HT16]
  - but no proof of security against high-order attacks was provided by the authors.
  - We describe an attack in the proceedings: 3rd order attack that works for any number of shares  $n$ .

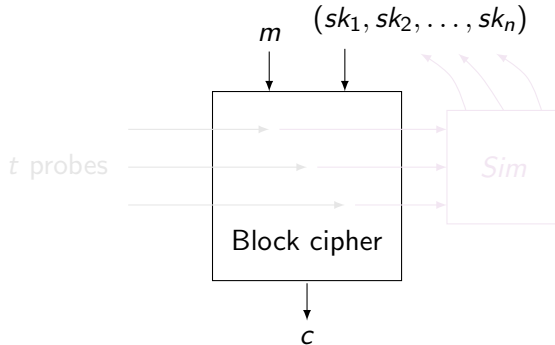


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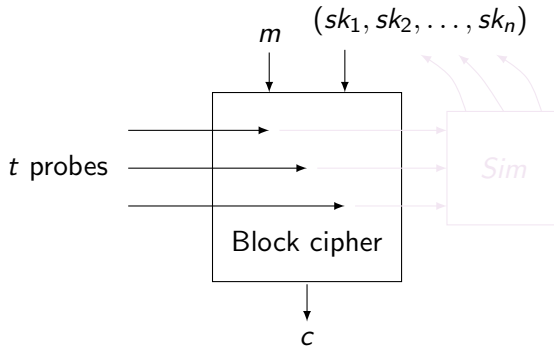
- Simulation framework of [ISW03]:



- Show that any  $t$  probes can be perfectly simulated from at most  $n - 1$  of the  $sk_i$ 's.
- Those  $n - 1$  shares  $sk_i$  are initially uniformly and independently distributed.
- $\Rightarrow$  the adversary learns nothing from the  $t$  probes, since he could perfectly simulate those  $t$  probes by himself.

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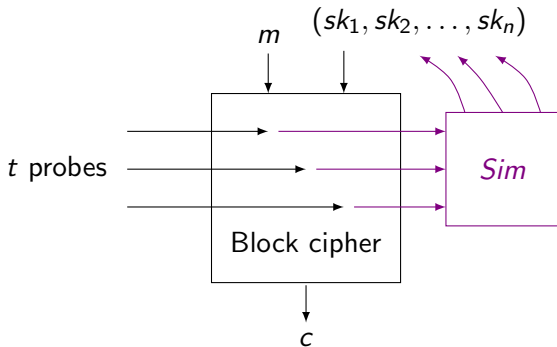
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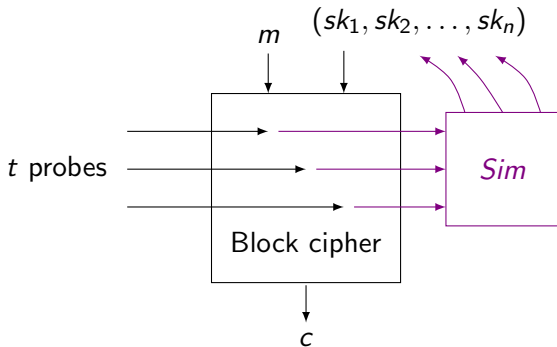
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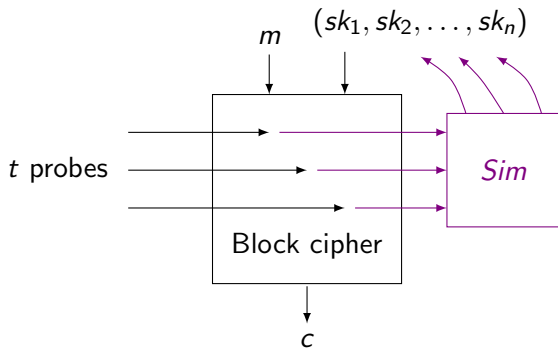
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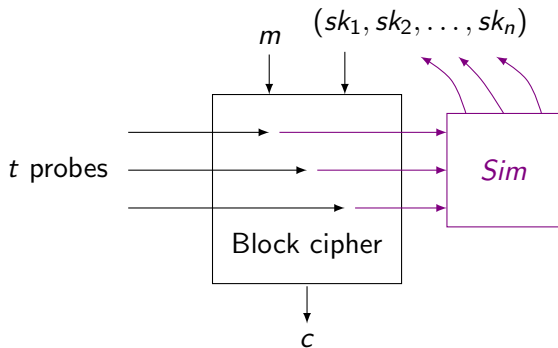
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## Security proofs for side-channel countermeasures

- Never publish a high-order masking scheme without a proof of security !
  - So many things can go wrong.
  - Many countermeasures without proofs have been broken in the past.
  - We have a poor intuition of high-order security.



## Goubin's original conversion algorithm

- Goubin's theorem: the function

$$\Psi(x, r) = (x \oplus r) - r \pmod{2^k}$$

is affine with respect to  $r$  over  $\mathbb{F}_2$ .

- This is surprising but true !
- Goubin's Boolean to arithmetic conversion algorithm:

$$\begin{aligned}x &= x_1 \oplus x_2 \\&= (x_1 \oplus x_2 - x_2) + x_2 \\&= \Psi(x_1, x_2) + x_2 \\&= [(x_1 \oplus \Psi(x_1, r \oplus x_2)) \oplus \Psi(x_1, r)] + x_2 \\&= A + x_2 \pmod{2^k}\end{aligned}$$

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## Our new algorithm: generalization of Goubin

- We start from

$$\begin{aligned}x &= x_1 \oplus \cdots \oplus x_n \\&= (x_1 \oplus x_2 \oplus \cdots \oplus x_n - x_2 \oplus \cdots \oplus x_n) + x_2 \oplus \cdots \oplus x_n \\&= \Psi(x_1, x_2 \oplus \cdots \oplus x_n) + x_2 \oplus \cdots \oplus x_n \\&= (n \wedge 1) \cdot x_1 \oplus \Psi(x_1, x_2) \oplus \cdots \oplus \Psi(x_1, x_n) + x_2 \oplus \cdots \oplus x_n \\&= z_1 \oplus \cdots \oplus z_{n-1} + x_2 \oplus \cdots \oplus x_n\end{aligned}$$

- We can apply the algorithm recursively on both terms with  $n - 1$  shares:

$$\begin{aligned}x &= A_1 + \cdots + A_{n-1} + B_1 + \cdots + B_{n-1} \\&= (A_1 + B_1) + \cdots + (A_{n-2} + B_{n-2}) + A_{n-1} + B_{n-1} \\&= D_1 + \cdots + D_{n-2} + D_{n-1} + D_n\end{aligned}$$

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$$\begin{aligned}x &= x_1 \oplus \cdots \oplus x_n \\&= (x_1 \oplus x_2 \oplus \cdots \oplus x_n - x_2 \oplus \cdots \oplus x_n) + x_2 \oplus \cdots \oplus x_n \\&= \Psi(x_1, x_2 \oplus \cdots \oplus x_n) + x_2 \oplus \cdots \oplus x_n \\&= (n \wedge 1) \cdot x_1 \oplus \Psi(x_1, x_2) \oplus \cdots \oplus \Psi(x_1, x_n) + x_2 \oplus \cdots \oplus x_n \\&= z_1 \oplus \cdots \oplus z_{n-1} + x_2 \oplus \cdots \oplus x_n\end{aligned}$$

- We can apply the algorithm recursively on both terms with  $n - 1$  shares:

$$\begin{aligned}x &= A_1 + \cdots + A_{n-1} + B_1 + \cdots + B_{n-1} \\&= (A_1 + B_1) + \cdots + (A_{n-2} + B_{n-2}) + A_{n-1} + B_{n-1} \\&= D_1 + \cdots + D_{n-2} + D_{n-1} + D_n\end{aligned}$$

- We obtain  $n$  arithmetic shares as required.

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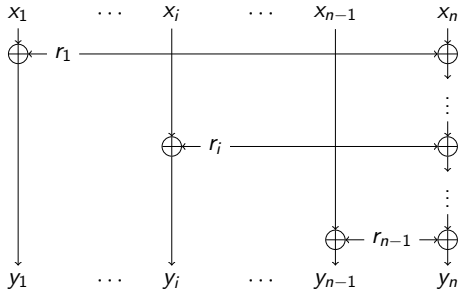
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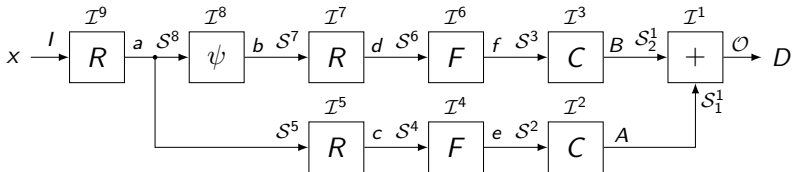
- We must add some intermediate mask refreshing, otherwise the algorithm would be insecure:





# Proof of Security in the ISW probing model

- We use the  $t$ -NI and  $t$ -SNI security definitions introduced by Barthe *et al.* in [BBD+16]
  - This enables to have a modular proof
  - We first analyse each gadget separately
  - We then compose the gadgets



- See the proof in the ePrint version of the paper.

## Operation count

- Operation count for Boolean to arithmetic conversion algorithms, with  $n = t + 1$  shares.

| <b>B → A conversion</b> | Security order $t$ |       |       |       |        |        |        |         |
|-------------------------|--------------------|-------|-------|-------|--------|--------|--------|---------|
|                         | 1                  | 2     | 3     | 4     | 6      | 8      | 10     | 12      |
| Goubin [Gou01]          | 7                  |       |       |       |        |        |        |         |
| Hutter-Tunstall [HT16]  |                    | 31    |       |       |        |        |        |         |
| CGV, 32 bits [CGV14]    |                    | 2 098 | 3 664 | 7 752 | 14 698 | 28 044 | 39 518 | 56 344  |
| Our algorithm           |                    | 55    | 155   | 367   | 1 687  | 7 039  | 28 519 | 114 511 |

- For small orders  $t$ , our algorithm is one order of magnitude more efficient than [CGV14].

# Conclusion

- We have described a new high-order Boolean to arithmetic conversion algorithm.
  - Extension of Goubin's first-order algorithm
  - Provably secure in the ISW probing model
- Complexity:  $\mathcal{O}(2^n)$  for  $n$  shares, independent of the register size  $k$ .
  - Instead of  $\mathcal{O}(n^2 \cdot k)$  in [CGV14]
  - but one order of magnitude faster for small  $n$
- Open problem: can we do better than  $\mathcal{O}(2^n)$  ?

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