Radboud University

Changing of the Guards

Joan Daemen

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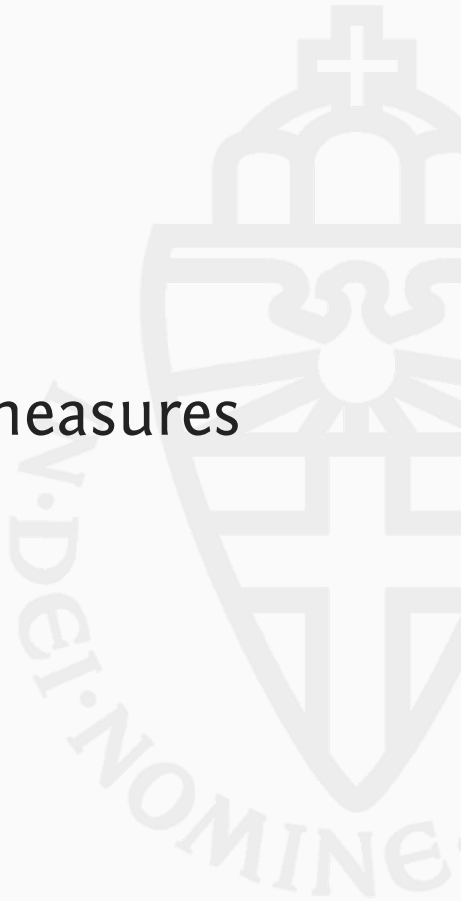
Radboud University

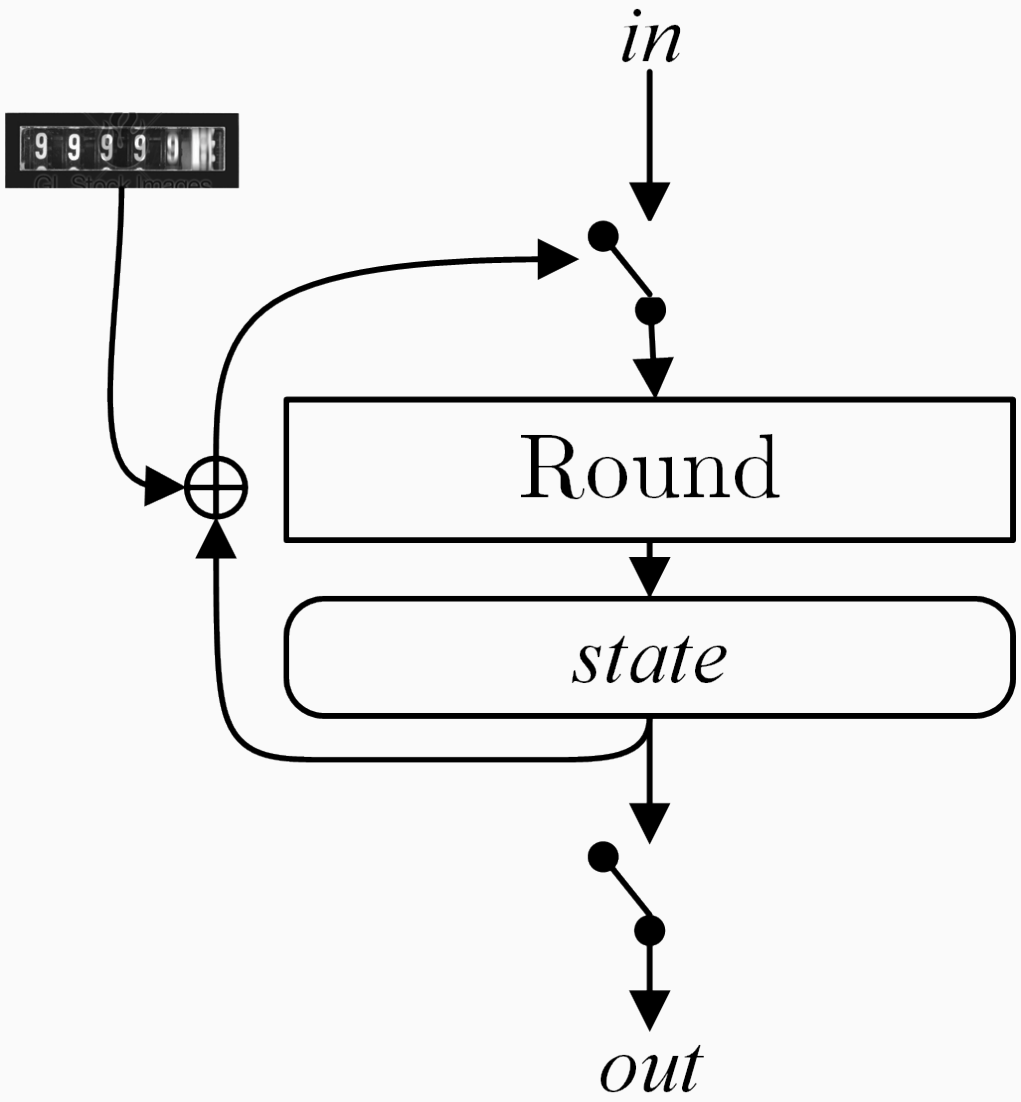
STMicroelectronics

Disclaimer

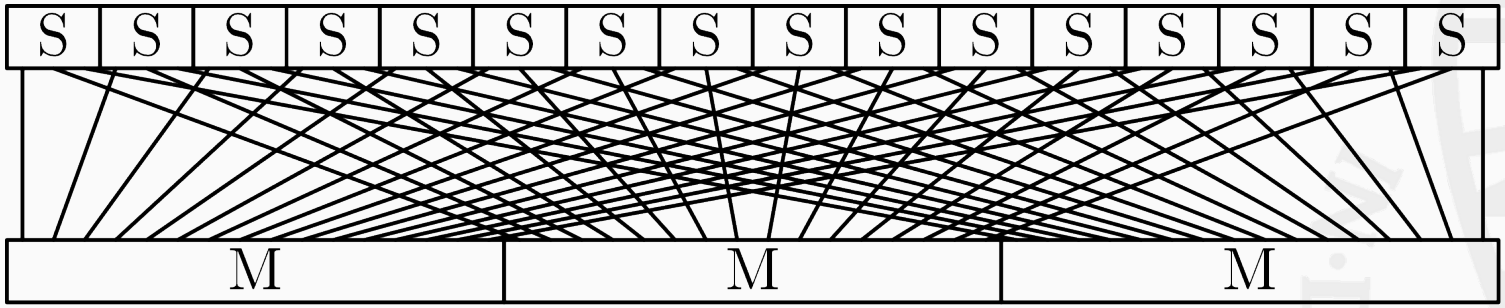
This is not a talk about higher-order countermeasures



Iterative cryptographic permutation



Three-stage round function: wide trail



Nonlinear layer χ

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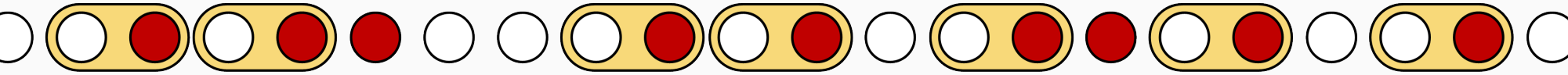

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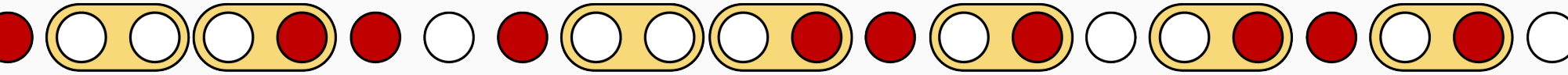

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Invertible for odd length n

Used in KETJE, KEYAK, KECCAK ($n = 5$)

RADIOGATUN ($n = 19$), PANAMA ($n = 17$), BASEKING,

3-WAY ($n = 3$), SUBTERRANEAN, CELLHASH ($n = 257$)

[Daemen, Govaerts, Vandewalle, WIC Benelux 1991]

Masking of χ as DPA/DEMA countermeasure

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$$\begin{aligned} a_0 &\leftarrow a_0 + (a_1 + 1)a_2 + a_1 b_2 \\ b_0 &\leftarrow b_0 + (b_1 + 1)b_2 + b_1 a_2 \end{aligned}$$

[Daemen, Peeters, Van Assche, FSE 2000]

χ': a three-share masking of χ

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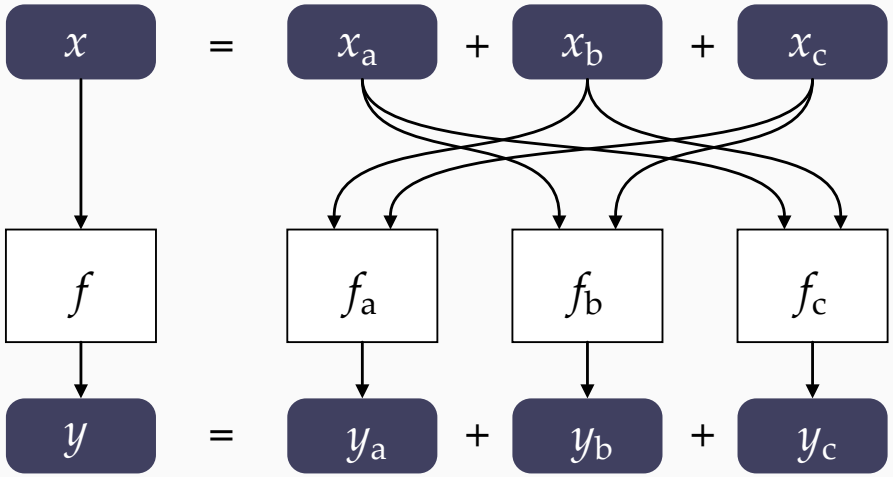
[Bertoni, Daemen, Peeters, Van Assche, 2nd SHA-3, 2010]

Threshold masking schemes [Nikova, Rijmen, Rechberger, Schläffer, '06 - '08]



Scheme at the right computes f securely against 1st order DPA if:

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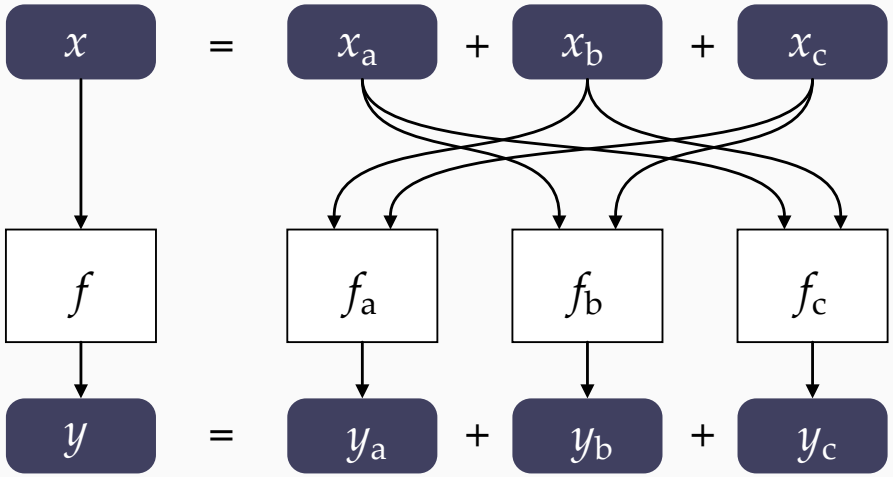

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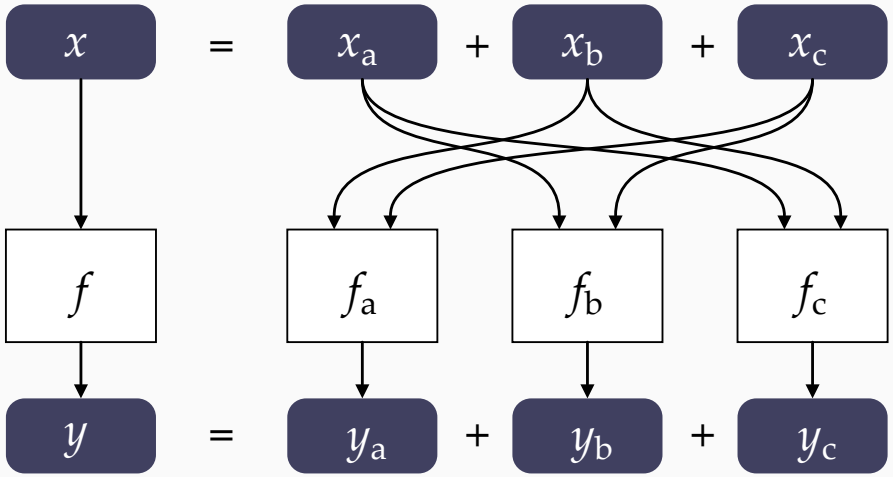

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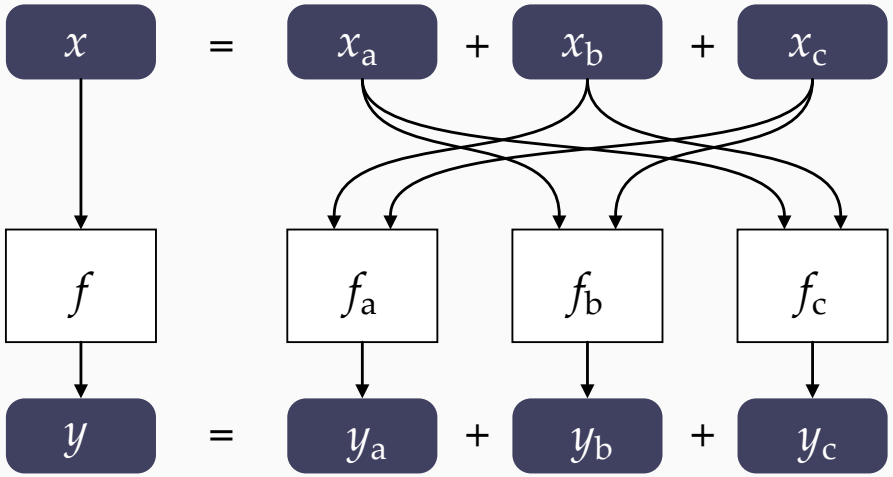

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- (x_a, x_b, x_c) is a uniform sharing of x:
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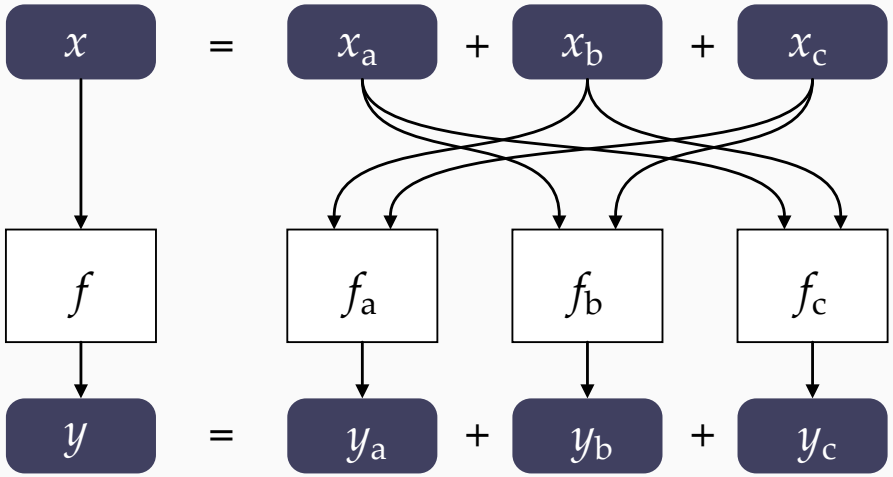

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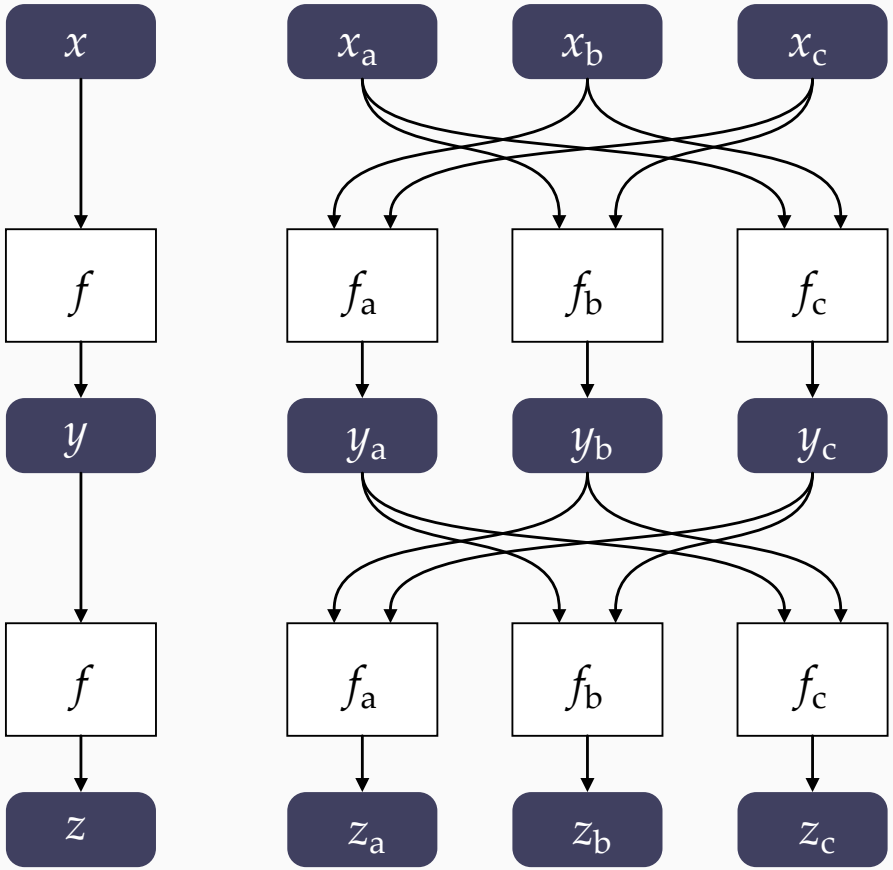
  - all values (x_a, x_b, x_c) with $x_a + x_b + x_c = x$ equiprobable
  - $x = 0 : (x_a, x_b, x_c) \in \{(0,0,0)(1,1,0)(1,0,1)(0,1,1)\}$
  - $x = 1 : (x_a, x_b, x_c) \in \{(1,1,1)(0,0,1)(0,1,0)(1,0,0)\}$

Uniformity of a threshold masking scheme



- Sharing (f_a, f_b, f_c) of f is called *uniform* if it preserves uniformity

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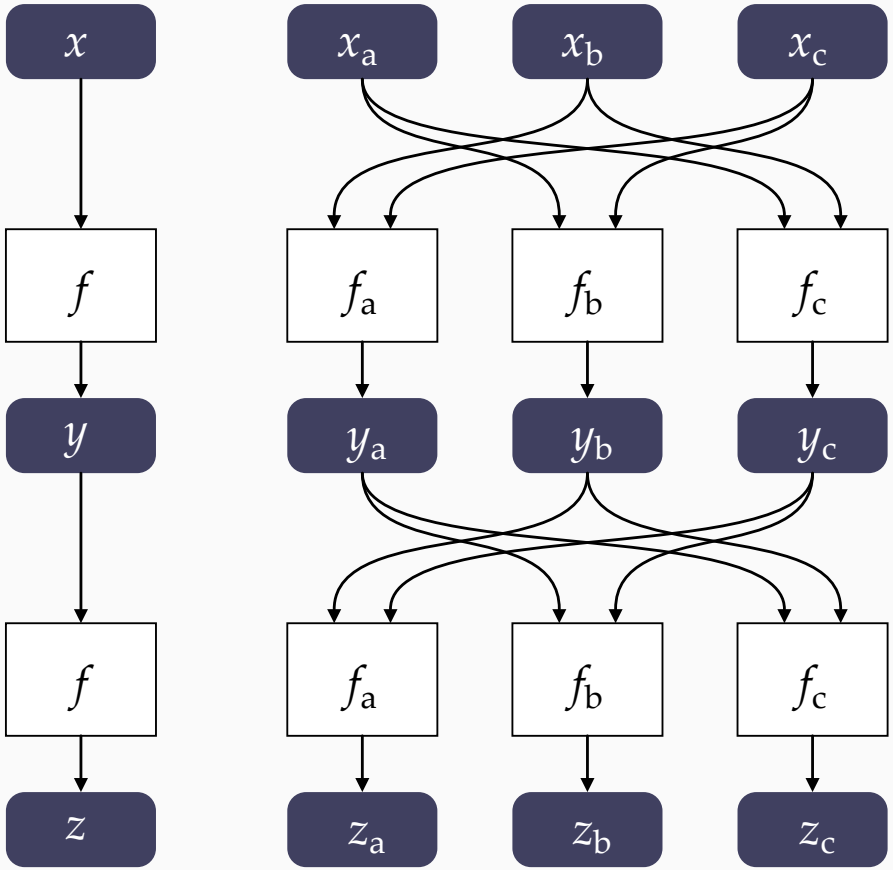

- Sharing (f_a, f_b, f_c) of f is called *uniform* if it preserves uniformity
- If f is invertible, for (f_a, f_b, f_c) uniformity $=$ invertibility

Back to χ'

$$
\begin{aligned}
a_0 &\leftarrow b_0 + (b_1 + 1)b_2 + b_1c_2 + b_2c_1 \\
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Is this a secure threshold masking scheme of χ?

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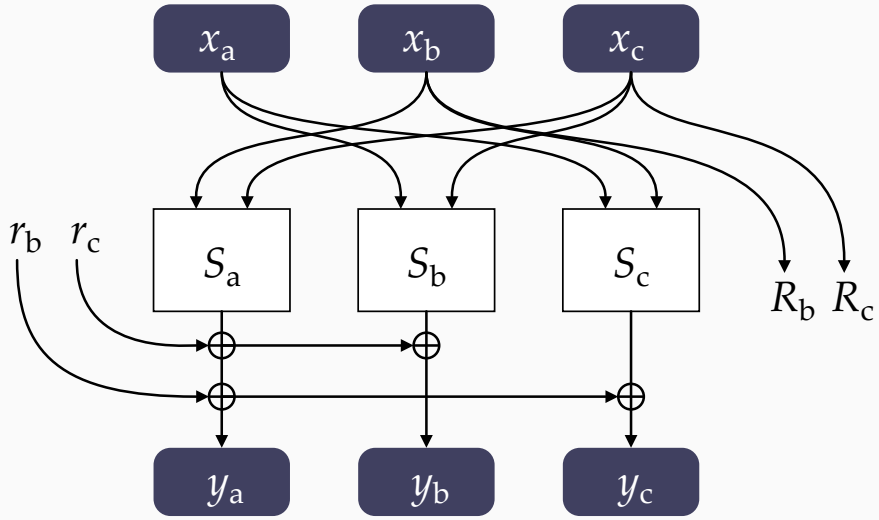
In general, for many S-boxes:

- no uniform $d + 1$-share threshold schemes are known
- it is an active research area to find the best compromise

An out-of-the-box approach to achieving uniformity



(1) Build (S_a, S_b, S_c): a correct and incomplete sharing of S

- straightforward and generalizes to $d+1$ shares for $d > 2$

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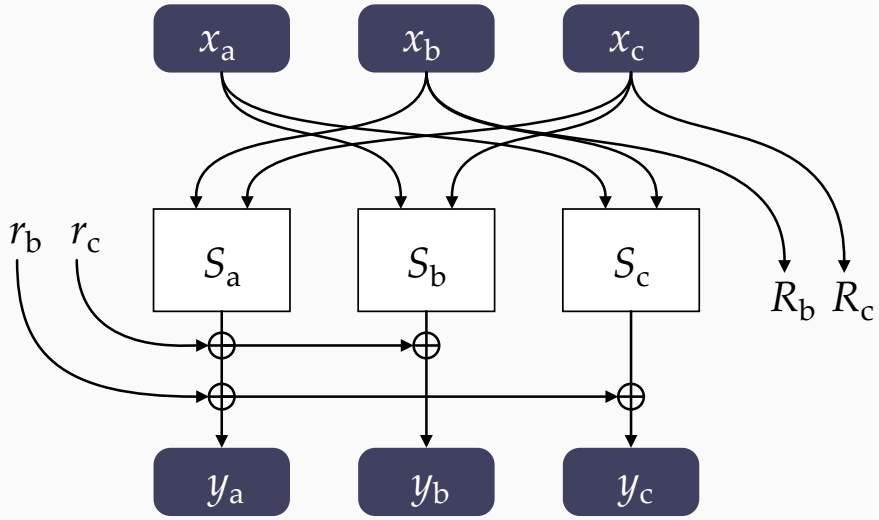

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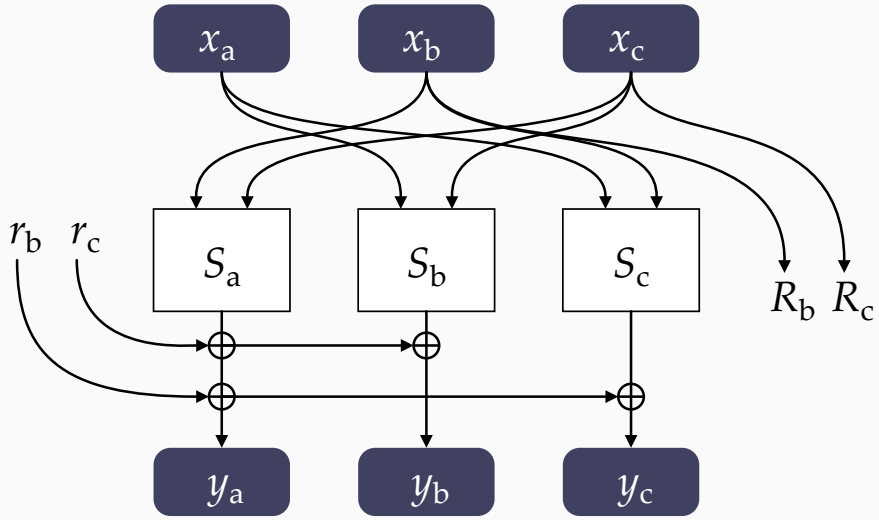

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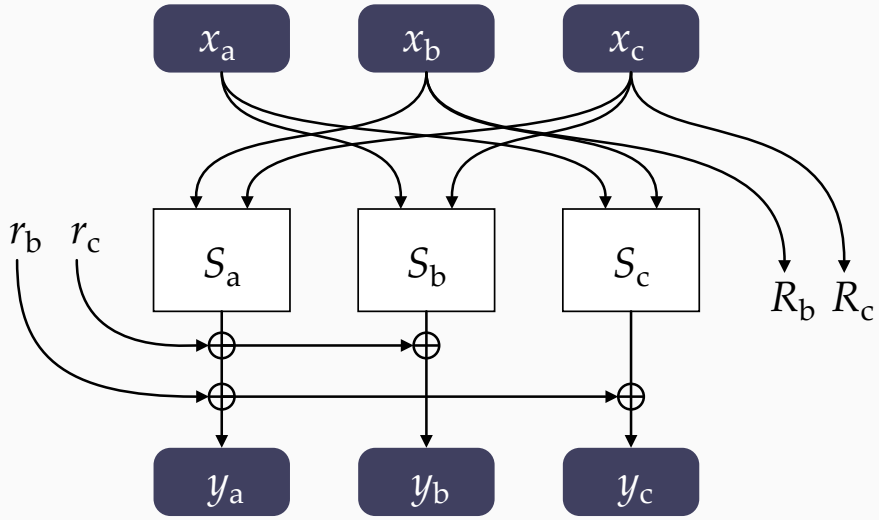

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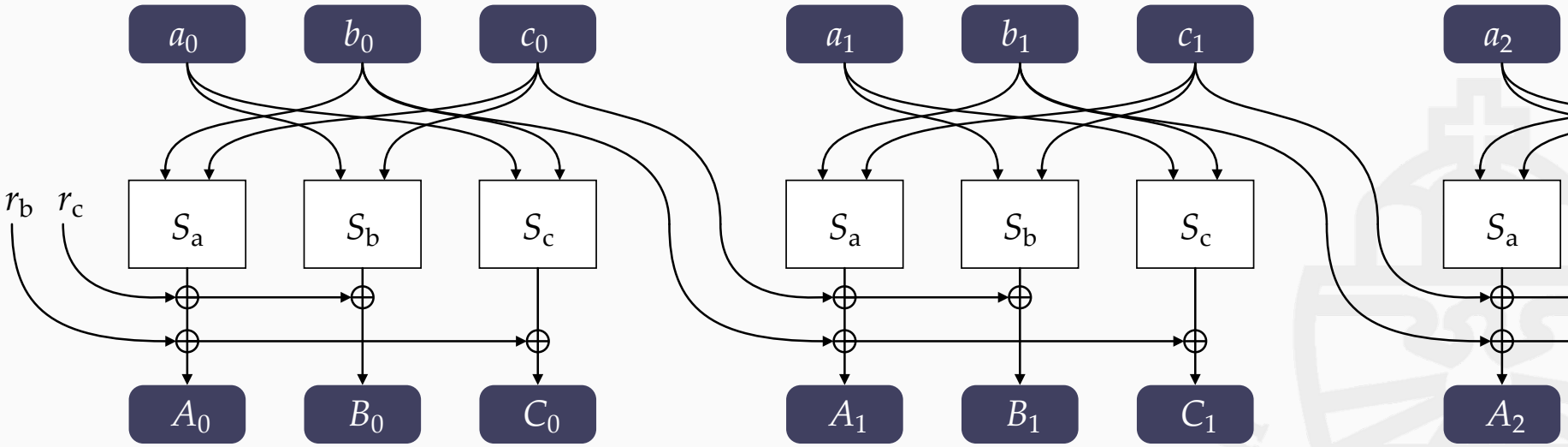

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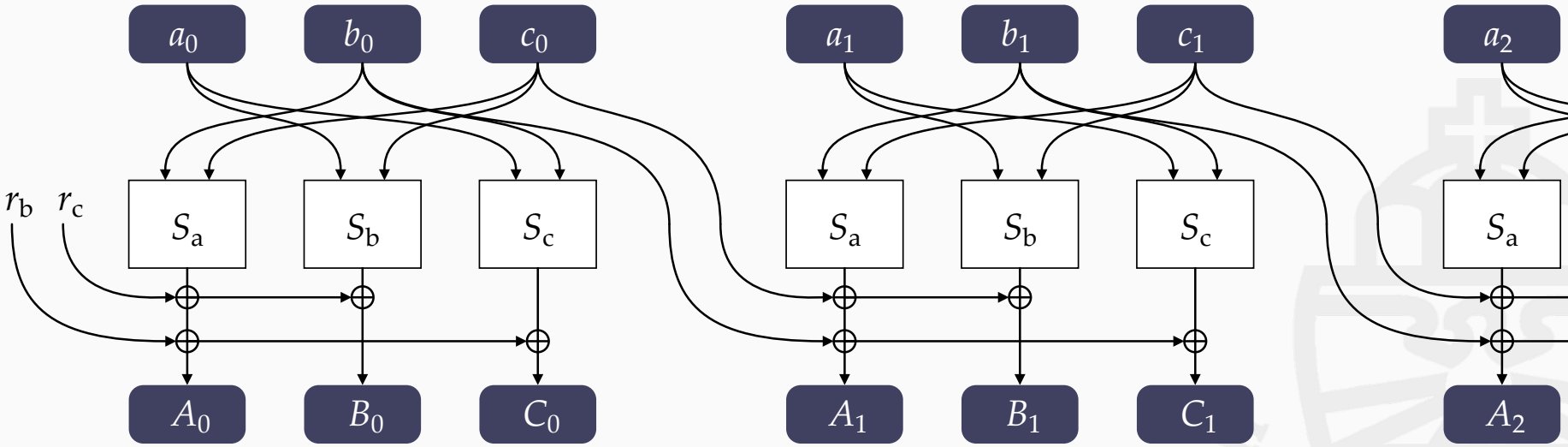

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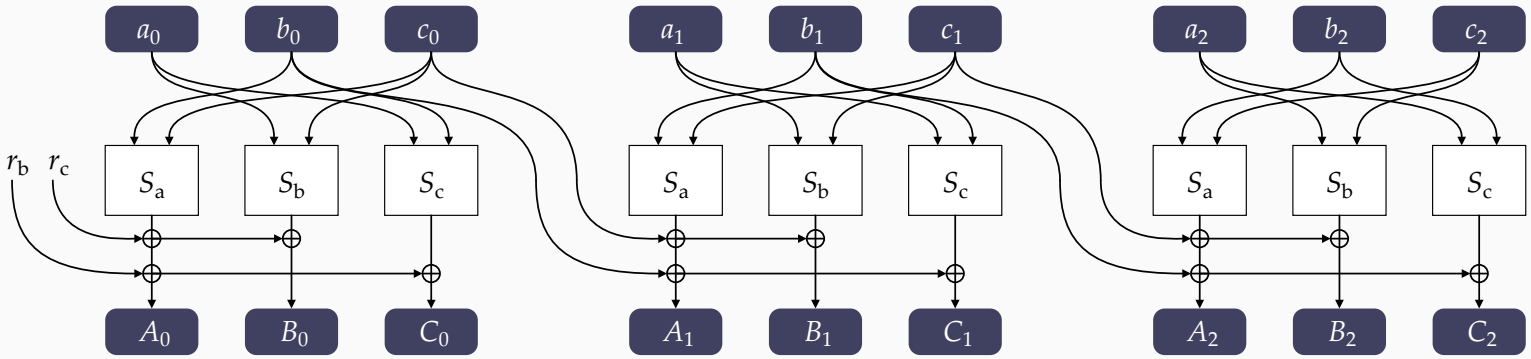
But where does leftmost (r_b, r_c) come from?

Attempt 1: injecting fresh randomness



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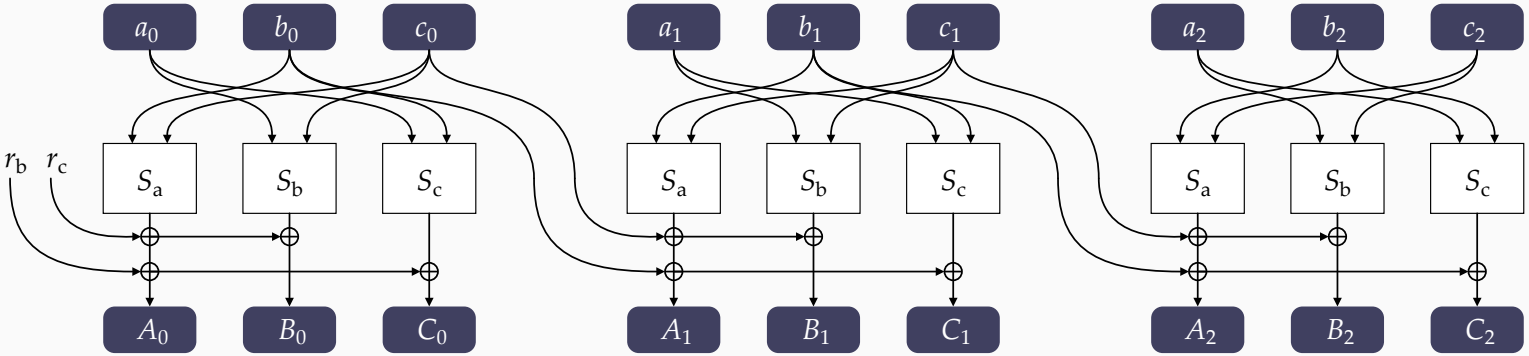

- (r_b, r_c) are generated freshly every round

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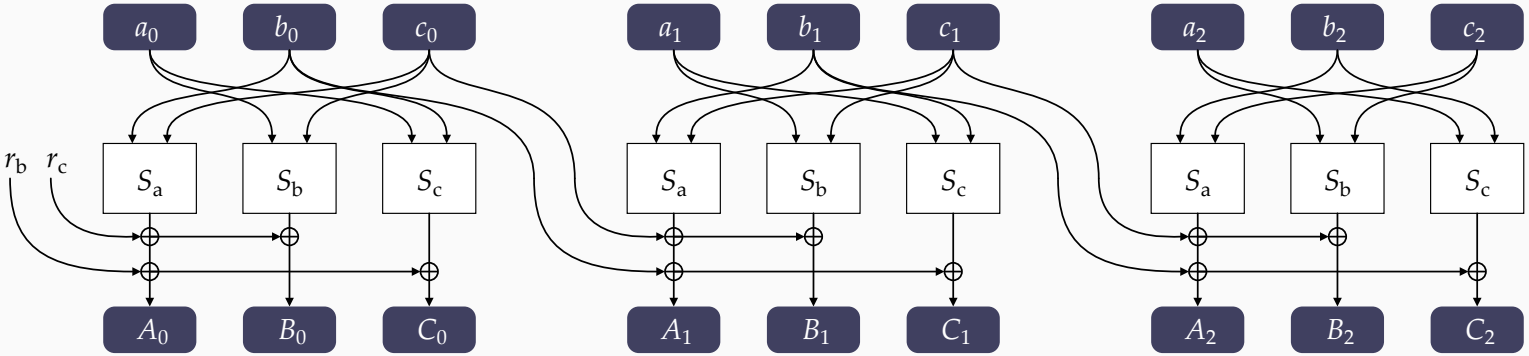

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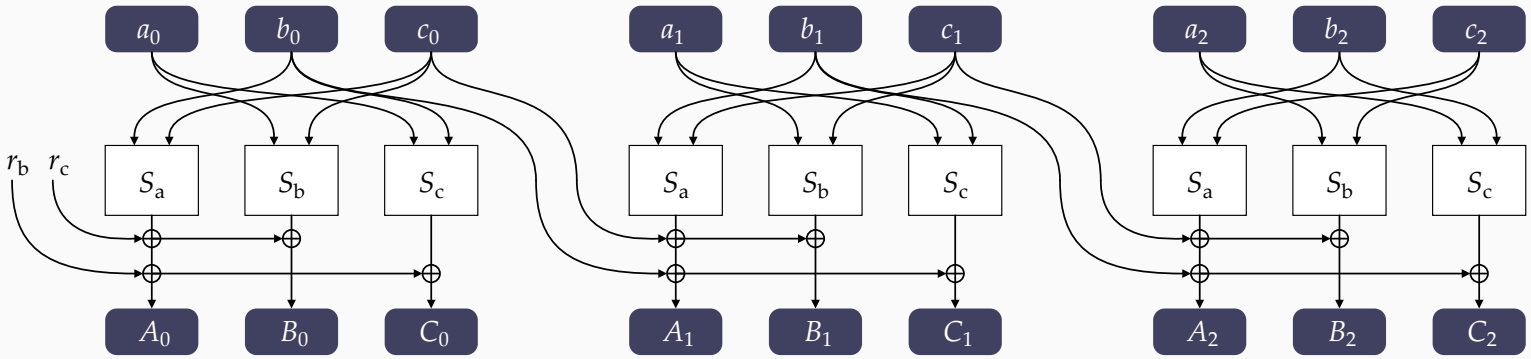

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  - real-world: requires random generation during operation
  - academic: no uniform sharing is obtained

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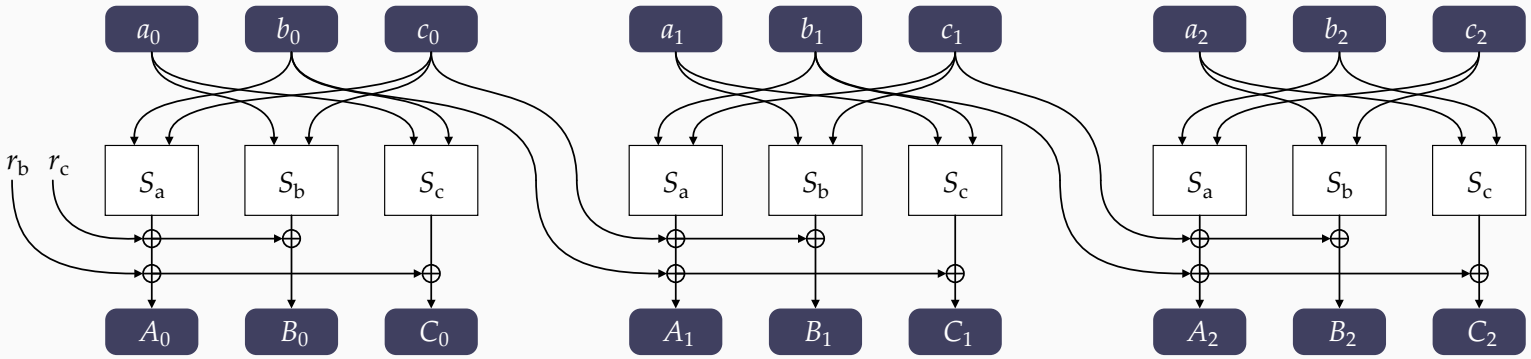

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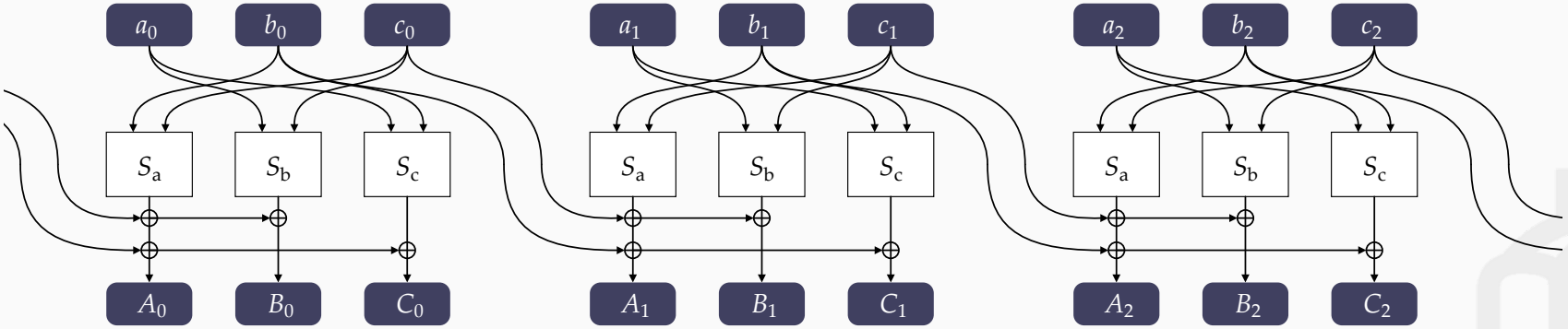
[Bilgin, Daemen, Nikova, Nikov, Rijmen, Van Assche, Cardis '13]

Attempt 2: cycling randomness



- S-boxes are arranged in circle: $(r_b, r_c) = (R_b, R_c)$

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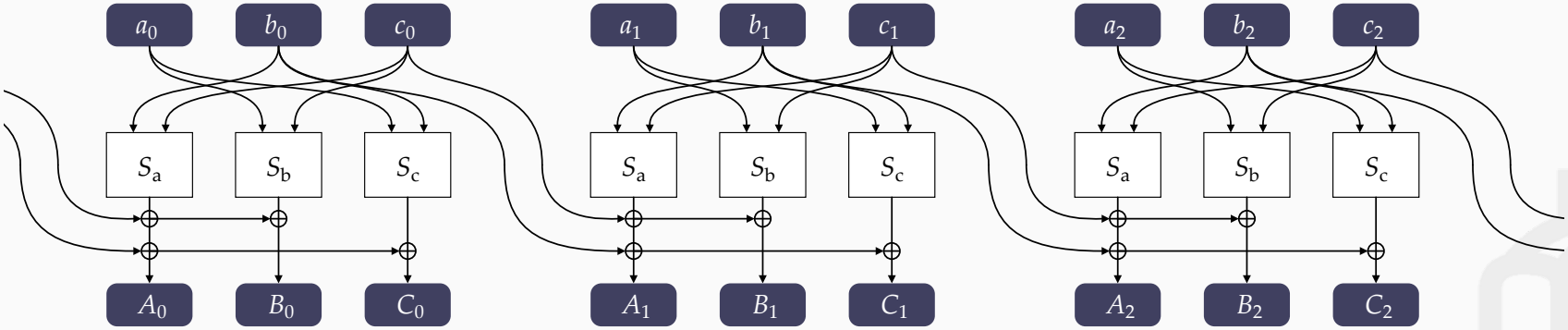

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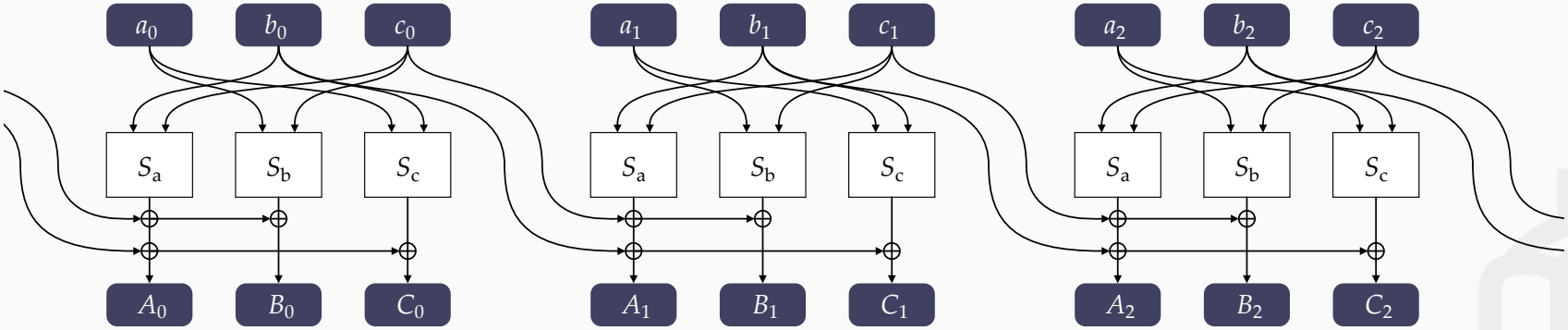
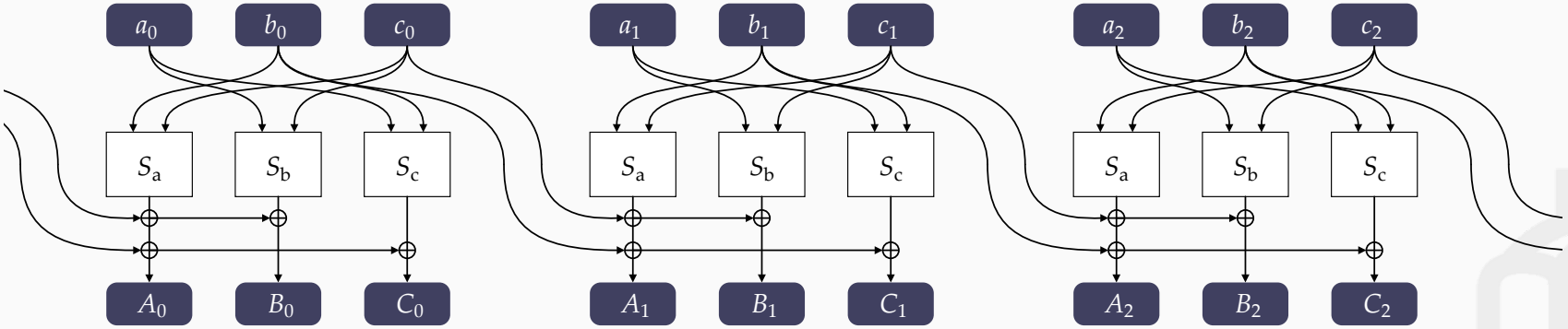


- S-boxes are arranged in circle: $(r_b, r_c) = (R_b, R_c)$
- No more need for generating randomness during operation
- The remaining amount of non-uniformity is negligible
- Downsides:
  - real-world: hard to explain why that is the case ...
  - academic: it is simply not uniform!

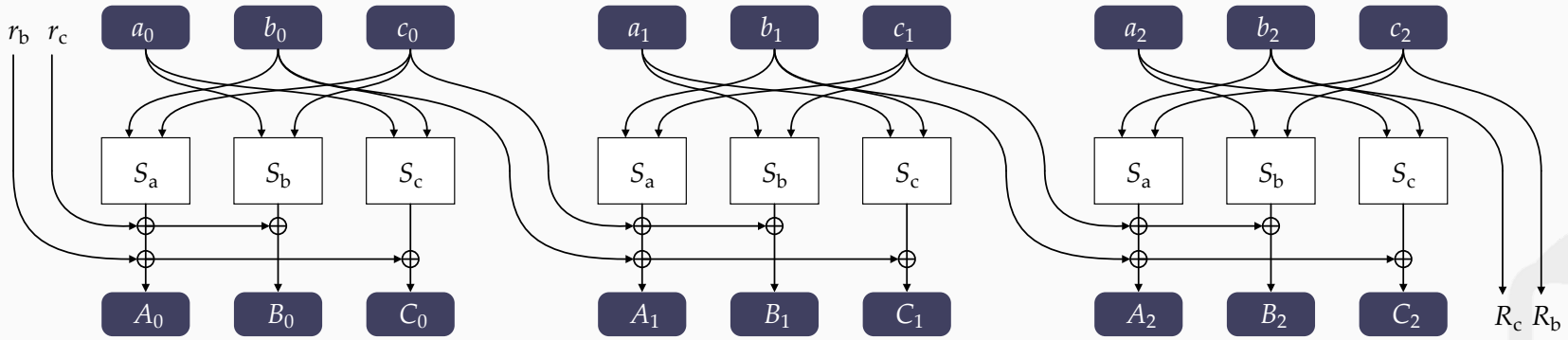
Attempt 2: cycling randomness



- S-boxes are arranged in circle: $(r_b, r_c) = (R_b, R_c)$
- No more need for generating randomness during operation
- The remaining amount of non-uniformity is negligible
- Downsides:
  - real-world: hard to explain why that is the case ...
  - academic: it is simply not uniform!

I presented this at [Shonan, Sep.'14] [ESC, Jan.'15] [TI Day, May'15]

Attempt 3: recycling randomness



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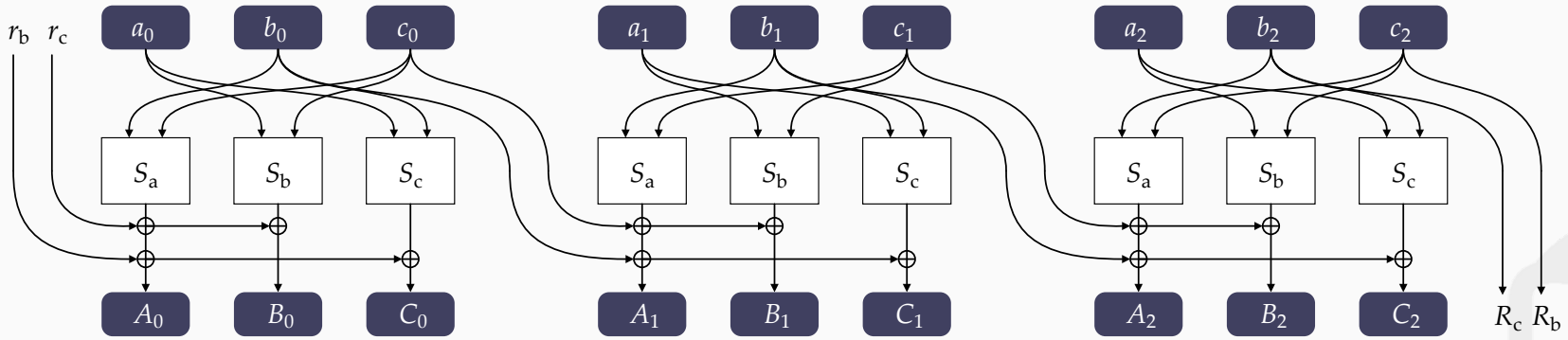

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- input Guards (r_b, r_c) are previous-round output Guards (R_b, R_c)

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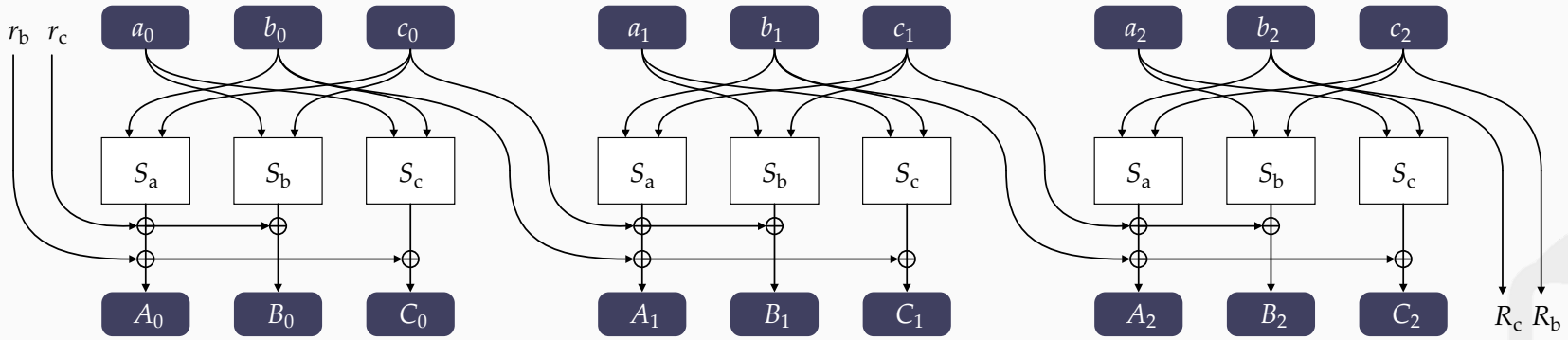

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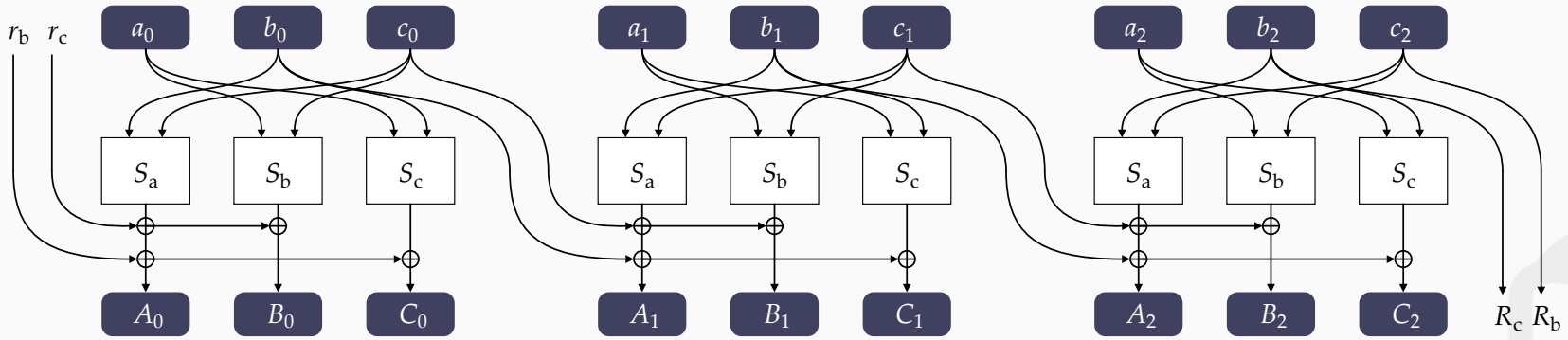

- We make (R_b, R_c) part of the shared state: the Guards
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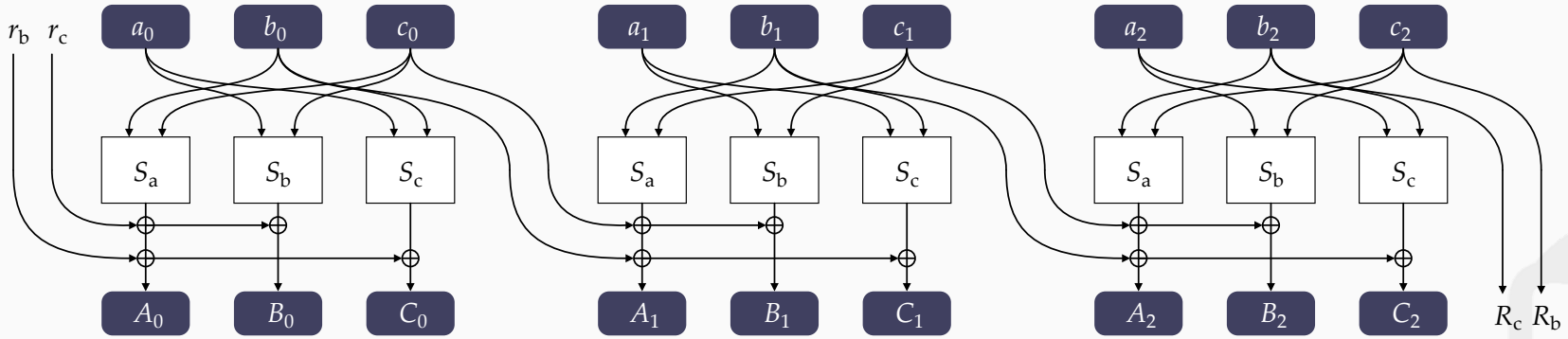
- Achieves uniformity if S-box is invertible
- Cost:
  - 4 additional XORs per native bit
  - shared state extended by $2n$ additional bits (for n-bit S-box)

Proof of uniformity



Computing (a, b, c) and (r_b, r_c) from (A, B, C) and (R_b, R_c)

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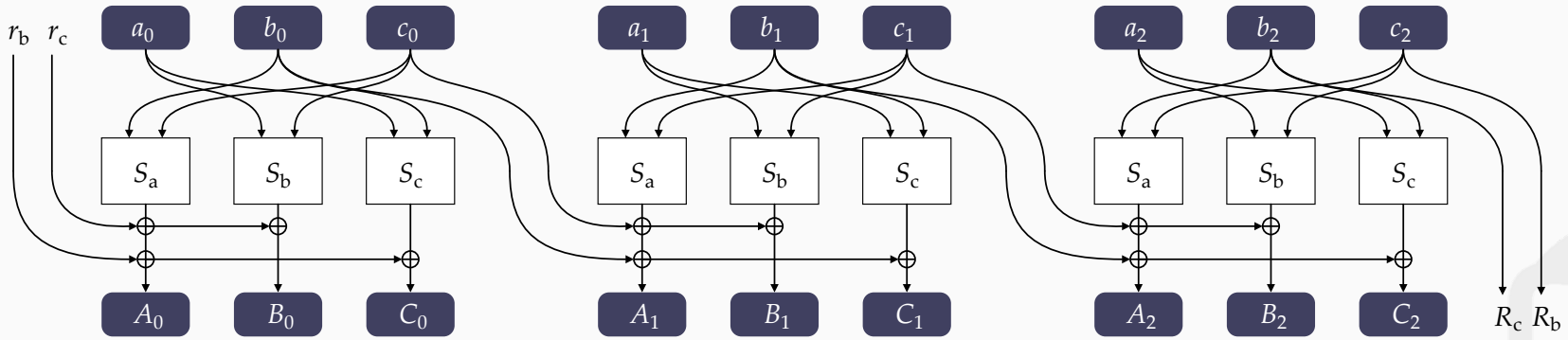

Computing (a, b, c) and (r_b, r_c) from (A, B, C) and (R_b, R_c)

- Initial step: $b_2 \leftarrow R_c$ and $c_2 \leftarrow R_b$
- Iteration: compute (a_i, b_{i-1}, c_{i-1}) from (A_i, B_i, C_i) and (b_i, c_i)
  - $a_i = S^{-1}(A_i + B_i + C_i) + b_i + c_i$
  - $b_{i-1} = S_c(a_i, b_i) + C_i$
  - $c_{i-1} = S_b(a_i, b_i) + B_i$
- Final step: $r_b \leftarrow b_{-1}$ and $r_c \leftarrow c_{-1}$
- Invertibility implies uniformity: QED

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- Assume we know
  - bits of (a, b, c) with indices 0 and 1
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A_m &= b_m + (b_0 + 1)b_1 + b_0 c_1 + b_1 c_0 \\
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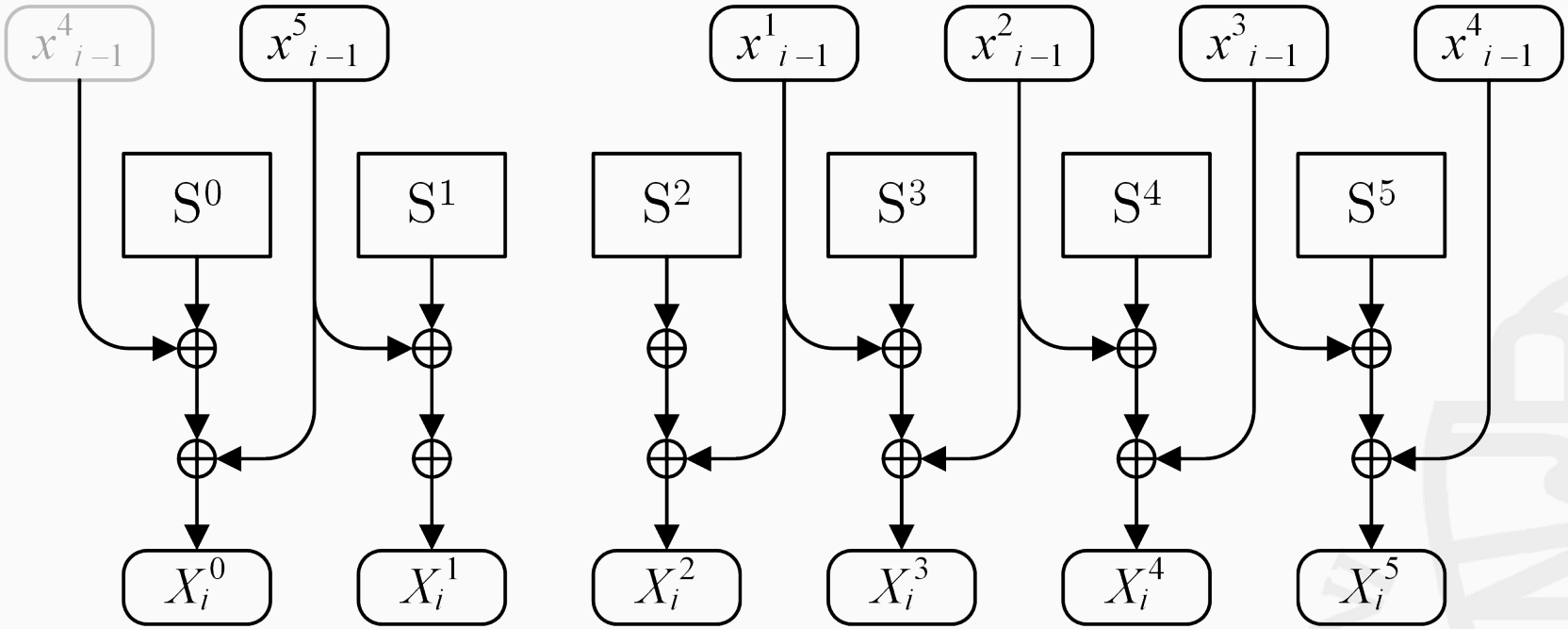
- This allows us to
  - reduce output masking to bits with indices in 0 and 1
  - shrink r_b and r_c to two bits each

Generalization for invertible n-bit S-box of degree d



- Guards: d shares of n bits
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- Total cost (worst case)

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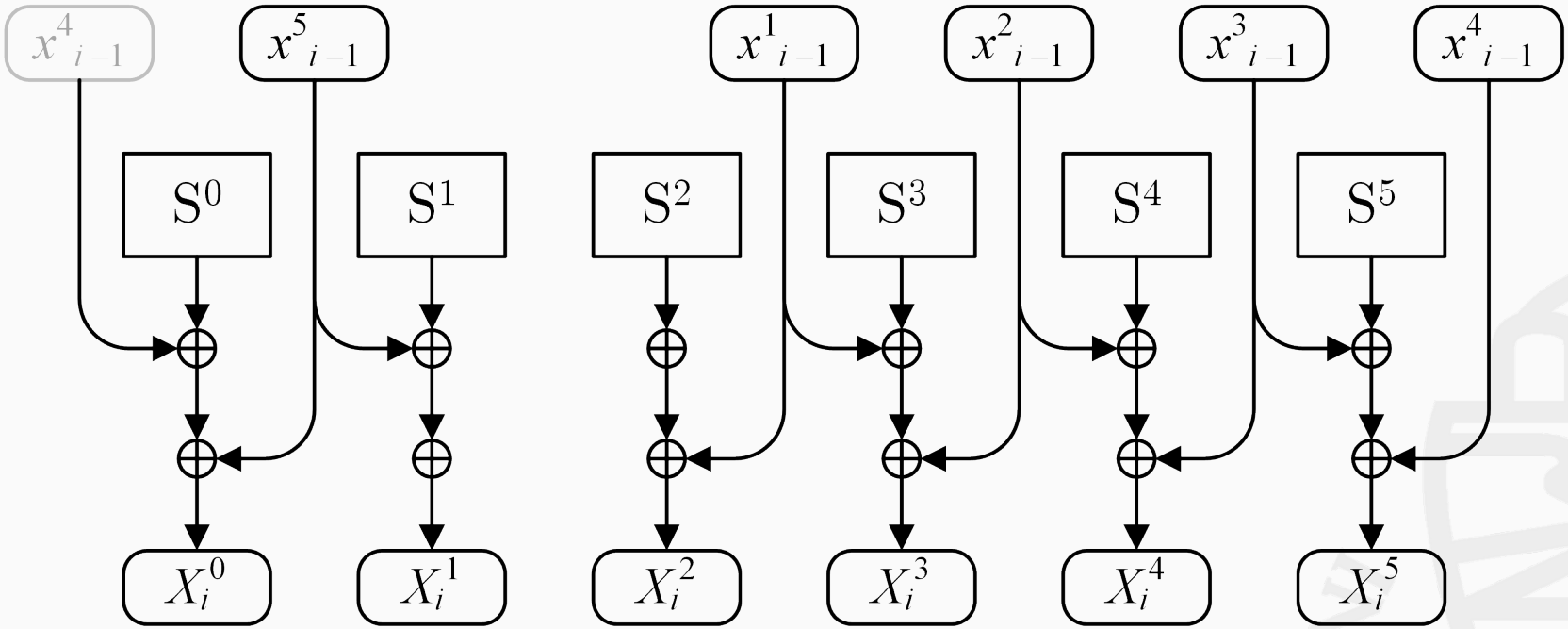

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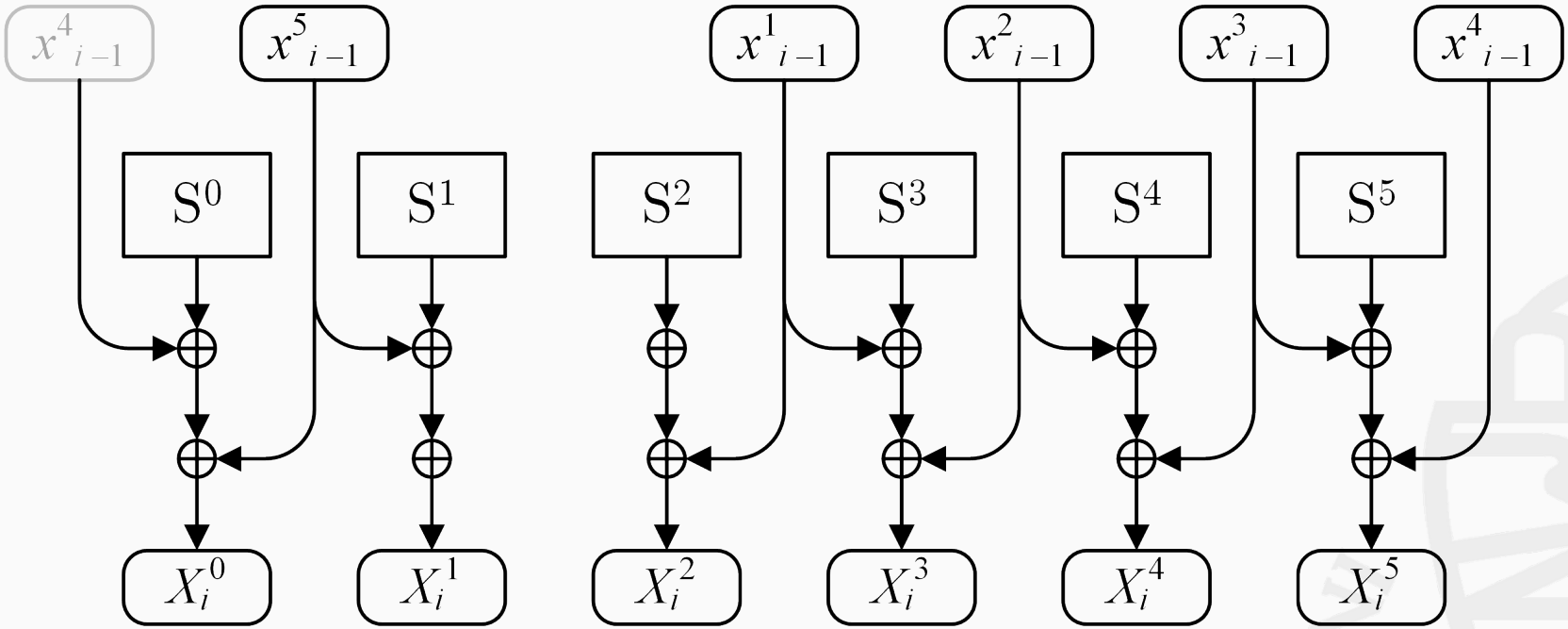

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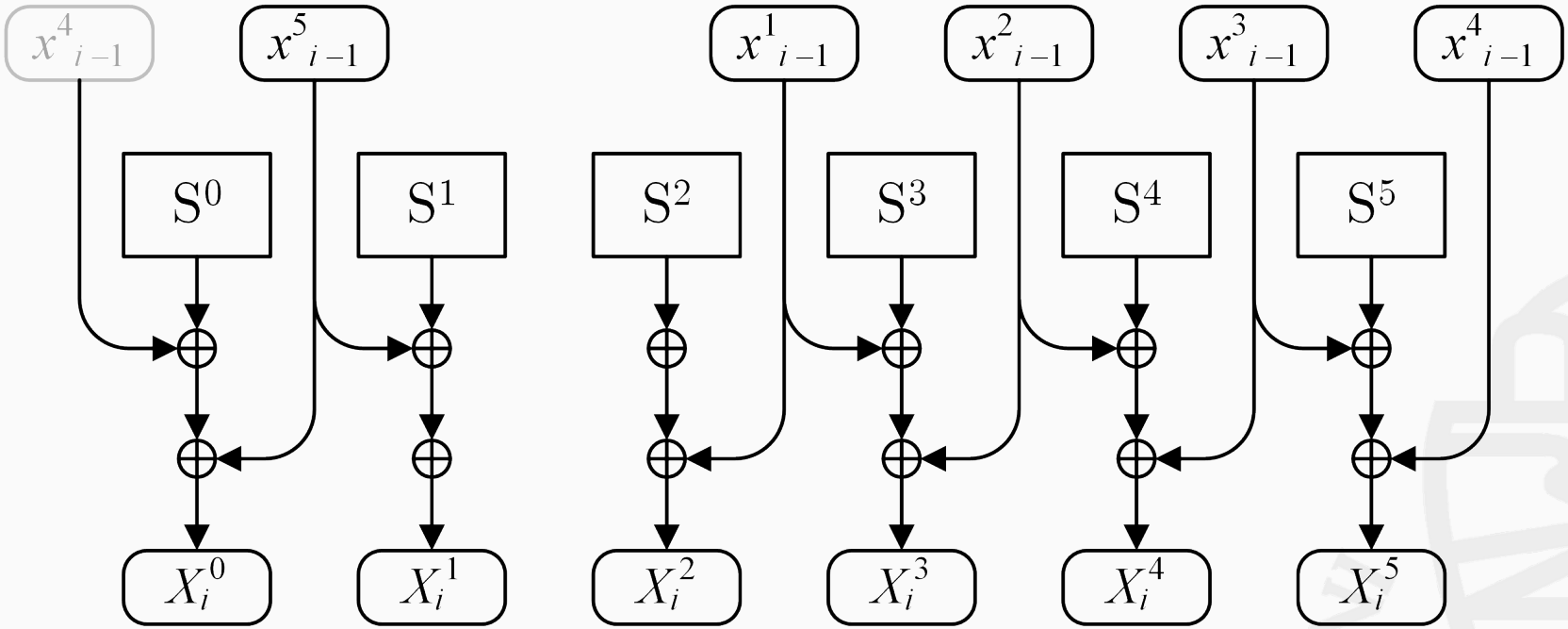

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- Cost is reduced if shared S-box has multi-transformation property

Conclusions

- Solution for achieving uniformity for invertible S-box layers
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Thanks for your attention!

Q?