



# High Order Masking of Look-up Tables with Common Shares

J-S.Coron, F.Rondepierre, R.Zeitoun



12th September 2018



# Outline

---

Outline

- 1 Introduction
  - 1st Order Solution
  - Higher Order Masking of Look-Up Tables

- 2 Higher Order: Optimizations

- 3 Conclusion

12th September 2018



# Table of Contents

---

Introduction

- ① Introduction
  - 1st Order Solution
  - Higher Order Masking of Look-Up Tables

② Higher Order: Optimizations

③ Conclusion

12th September 2018



## Sharing Principle

- Given a sensitive data  $x$
- Given  $t$  random values  $x_1, \dots, x_t$
- Let  $x_0$  be such that:

$$x = \bigoplus_{i=0}^t x_i$$

- $(x_0, \dots, x_t)$  is a sharing of  $x$  secure at order  $t$



## The problematic

- Given sensitive data  $x$
- Given a known table  $S$
- How to compute securely :

$$x \mapsto S(x)$$



## The problematic

- Given sensitive data  $x$
- Given a known table  $S$
- How to compute securely for  $\ell$  evaluations:

$$x^{(\ell)} \mapsto S(x^{(\ell)})$$



# 1st Order

---

## Secure at 1st Order

The  $\ell$ -th evaluation of  $S$  is:

$$x^{(\ell)} = (x_0^{(\ell)}, m) \mapsto S(x^{(\ell)}) = (y_0^{(\ell)}, m)$$



## Masked SBox Construction

$$T = \left\{ \begin{array}{c} S(0 \oplus m) \oplus m \\ \vdots \\ S((2^k - 1) \oplus m) \oplus m \end{array} \right\}$$

## Masked SBox Evaluation

$$S(x) = (T(x_0), m)$$





## Secure at Higher Order (Coron EUROCRYPT'14)

The  $\ell$ -th evaluation of  $S$  is:

$$x^{(\ell)} = (x_0^{(\ell)}, x_1^{(\ell)}, \dots, x_{2t}^{(\ell)}) \mapsto S(x^{(\ell)}) = (y_0^{(\ell)}, y_1^{(\ell)}, \dots, y_{2t}^{(\ell)})$$



## Higher Order

---

### Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$\begin{Bmatrix} S(0) & 0 & 0 & 0 \\ S(1) & 0 & 0 & 0 \\ S(2) & 0 & 0 & 0 \\ S(3) & 0 & 0 & 0 \end{Bmatrix}$$



## Higher Order

---

### Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$\begin{Bmatrix} S(2) & 0 & 0 & 0 \\ S(3) & 0 & 0 & 0 \\ S(0) & 0 & 0 & 0 \\ S(1) & 0 & 0 & 0 \end{Bmatrix}$$



## Higher Order

### Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$\left\{ \begin{array}{l} S(2) \oplus 3 \quad 1 \quad 2 \quad 0 \\ S(3) \oplus 1 \quad 0 \quad 0 \quad 1 \\ S(0) \oplus 0 \quad 2 \quad 3 \quad 1 \\ S(1) \oplus 0 \quad 0 \quad 0 \quad 0 \end{array} \right\}$$



## Higher Order

### Example at 3rd Order

$$x = 2 = (0, 1, \mathbf{1}, 2)$$

$$\left\{ \begin{array}{l} S(2) \oplus 3 \\ S(3) \oplus 1 \\ S(0) \oplus 0 \\ S(1) \oplus 0 \end{array} \begin{array}{l} 1 \\ 0 \\ 2 \\ 0 \end{array} \begin{array}{l} 2 \\ 0 \\ 3 \\ 0 \end{array} \begin{array}{l} 0 \\ 1 \\ 1 \\ 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} S(3) \oplus 1 \\ S(2) \oplus 3 \\ S(1) \oplus 0 \\ S(0) \oplus 0 \end{array} \begin{array}{l} 0 \\ 1 \\ 0 \\ 2 \end{array} \begin{array}{l} 0 \\ 2 \\ 0 \\ 3 \end{array} \begin{array}{l} 1 \\ 0 \\ 0 \\ 1 \end{array} \right\}$$



## Higher Order

---

### Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$\left\{ \begin{array}{l} S(3) \oplus 1 \quad 1 \quad 2 \quad 2 \\ S(2) \oplus 1 \quad 0 \quad 2 \quad 3 \\ S(1) \oplus 3 \quad 2 \quad 1 \quad 0 \\ S(0) \oplus 0 \quad 1 \quad 0 \quad 1 \end{array} \right\}$$



## Higher Order

### Example at 3rd Order

$$x = 2 = (0, \mathbf{1}, 1, 2)$$

$$\left\{ \begin{array}{l} S(2) \oplus 1 \quad 0 \quad 2 \quad 3 \\ S(3) \oplus 1 \quad 1 \quad 2 \quad 2 \\ S(0) \oplus 0 \quad 1 \quad 0 \quad 1 \\ S(1) \oplus 3 \quad 2 \quad 1 \quad 0 \end{array} \right\}$$



## Higher Order

### Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$\left\{ \begin{array}{l} S(2) \oplus 2 \quad 2 \quad 2 \quad 2 \\ S(3) \oplus 1 \quad 1 \quad 2 \quad 2 \\ S(0) \oplus 1 \quad 1 \quad 1 \quad 1 \\ S(1) \oplus 3 \quad 3 \quad 3 \quad 3 \end{array} \right\}$$





# Higher Order

---

## Example at 3rd Order

$$x = 2 = (\mathbf{0}, 1, 1, 2)$$

$$S(2) \oplus 2 \quad 2 \quad 2 \quad 2$$



# Higher Order

---

## Example at 3rd Order

$$x = 2 = (0, 1, 1, 2)$$

$$S(2) \oplus 0 \quad 1 \quad 2 \quad 3$$



## Masked SBox Construction (Coron EUROCRYPT'14)

$$T^{(0)} = \left\{ \begin{array}{c} \text{sharing of } S(0) \\ \vdots \\ \text{sharing of } S(2^k - 1) \end{array} \right\}$$



## Masked SBox Construction (Coron EUROCRYPT'14)

$$T^{(1)} = \left\{ \begin{array}{l} \text{new sharing of } T^{(0)}(0 \oplus \mathbf{x}_{2t}) \\ \vdots \\ \text{new sharing of } T^{(0)}((2^k - 1) \oplus \mathbf{x}_{2t}) \end{array} \right\}$$



## Masked SBox Construction (Coron EUROCRYPT'14)

$$T^{(2)} = \left\{ \begin{array}{l} \text{new sharing of } T^{(1)}(0 \oplus \mathbf{x}_{2t-1}) \\ \vdots \\ \text{new sharing of } T^{(1)}((2^k - 1) \oplus \mathbf{x}_{2t-1}) \end{array} \right\}$$



### Masked SBox Construction (Coron EUROCRYPT'14)

$$T^{(2t)} = \left\{ \begin{array}{l} \text{new sharing of } T^{(2t-1)}(0 \oplus \mathbf{x}_1) \\ \vdots \\ \text{new sharing of } T^{(2t-1)}((2^k - 1) \oplus \mathbf{x}_1) \end{array} \right\}$$



## Masked SBox Construction (Coron EUROCRYPT'14)

$$T^{(2t)} = \left\{ \begin{array}{c} \text{new sharing of } T^{(2t-1)}(0 \oplus \mathbf{x}_1) \\ \vdots \\ \text{new sharing of } T^{(2t-1)}((2^k - 1) \oplus \mathbf{x}_1) \end{array} \right\}$$

## Masked SBox Evaluation

$$S(x) = \text{new sharing of } T^{(t)}(x_0)$$



# Table of Contents

---

Higher Order:  
Optimizations

- ① Introduction
  - 1st Order Solution
  - Higher Order Masking of Look-Up Tables

- ② Higher Order: Optimizations

- ③ Conclusion





## Our Contributions

- Security proof at order  $t$  with  $n = t + 1$  shares instead of  $n = 2t + 1$  shares (t-sni formalism)
  - Saves a factor 4 (running time)
- A variant with increasing number of output shares
  - Saves a factor 2 (running time)
- Adapt the common shares technique for multiple SBox evaluations
  - Saves a factor 2 (running time)



### Common Shares (CGPZ CHES16)

Two values  $a$  and  $b$  may be securely shared such that at most half of the shares are common:

$$(a_0, \dots, a_{\frac{t}{2}}, m_0, \dots, m_{\frac{t-1}{2}})$$

$$(b_0, \dots, b_{\frac{t}{2}}, m_0, \dots, m_{\frac{t-1}{2}})$$



## Look-Up Tables with Common Shares

---

### Secure at Higher Order

The  $\ell$ -th evaluation of  $S$  is:

$$(x_0^{(\ell)}, x_1^{(\ell)}, \dots, x_{\frac{t}{2}}^{(\ell)}, m_0, \dots, m_{\frac{t-1}{2}}) \mapsto S(x^{(\ell)}) = (y_0^{(\ell)}, y_1^{(\ell)}, \dots, y_t^{(\ell)})$$



## Look-Up Tables with Common Shares

---

Higher Order:  
Optimizations

### Masked SBox Construction (Common Table)

$$T^{(0)} = \left\{ \begin{array}{c} \text{sharing of } S(0) \\ \vdots \\ \text{sharing of } S(2^k - 1) \end{array} \right\}$$



## Look-Up Tables with Common Shares

### Masked SBox Construction (Common Table)

$$T^{(1)} = \left\{ \begin{array}{l} \text{new sharing of } T^{(0)}(0 \oplus \mathbf{m}_0) \\ \vdots \\ \text{new sharing of } T^{(0)}((2^k - 1) \oplus \mathbf{m}_0) \end{array} \right\}$$



## Look-Up Tables with Common Shares

### Masked SBox Construction (Common Table)

$$T^{(2)} = \left\{ \begin{array}{l} \text{new sharing of } T^{(1)}(0 \oplus \mathbf{m}_1) \\ \vdots \\ \text{new sharing of } T^{(1)}((2^k - 1) \oplus \mathbf{m}_1) \end{array} \right\}$$



## Look-Up Tables with Common Shares

### Masked SBox Construction (Common Table)

$$T^{\binom{t+1}{2}} = \left\{ \begin{array}{l} \text{new sharing of } T^{\binom{t-1}{2}}(0 \oplus \mathbf{m}_{\frac{t-1}{2}}) \\ \vdots \\ \text{new sharing of } T^{\binom{t-1}{2}}((2^k - 1) \oplus \mathbf{m}_{\frac{t-1}{2}}) \end{array} \right\}$$



## Look-Up Tables with Common Shares

### Masked SBox Construction (Common Table)

$$T^{(\frac{t+1}{2})} = \left\{ \begin{array}{c} \text{new sharing of } T^{(\frac{t-1}{2})}(0 \oplus \mathbf{m}_{\frac{t-1}{2}}) \\ \vdots \\ \text{new sharing of } T^{(\frac{t-1}{2})}((2^k - 1) \oplus \mathbf{m}_{\frac{t-1}{2}}) \end{array} \right\}$$

### Masked SBox Evaluation

- 1 Compute tables  $T^{(\frac{t+3}{2})}, \dots, T^{(t)}$  using shares  $x_1, \dots, x_{\frac{t}{2}}$
- 2 Evaluate using table  $T^{(t)}$ :

$$S(x) = \text{new sharing of } T^{(t)}(x_0)$$





## Performances



### AES

Higher Order:  
Optimizations

SBox Implementation	2	3	6
[RP10]	119	185	485
[Cor14]	2104	4413	17136
All optimizations	463	771	2767

Table: Software AES implementation, in thousand of clock cycles



### DES



SBox Implementation	2	3	6
[CGP+12]+[CRV14]	219	290	602
[Cor14]	491	907	3075
All optimizations	203	308	764

Table: Software DES implementation, in thousand of clock cycles



# Table of Contents

---

Conclusion

- ① Introduction
  - 1st Order Solution
  - Higher Order Masking of Look-Up Tables

- ② Higher Order: Optimizations

- ③ Conclusion

12th September 2018



### Conclusion

- Generalization of SBox recomputation, proven secure at any order
- Reduce the running time of common table by a factor of 2
- Reduce the running time by a factor of 8 (from Coron'14)
- Remaining task: build a proof to generalize common shares in outputs

$$(x_0^{(\ell)}, x_1^{(\ell)}, \dots, x_{\frac{t}{2}}^{(\ell)}, m_0, \dots, m_{\frac{t-1}{2}}) \mapsto S(x^{(\ell)}) = (y_0^{(\ell)}, y_1^{(\ell)}, \dots, y_{\frac{t}{2}}^{(\ell)}, m_0, \dots, m_{\frac{t-1}{2}})$$

- Correct solution for generic small SBox (e.g. DES)