Linear Repairing Codes and Side-Channel Attacks

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- Theory available to prove the security in (relatively) sound models *DucDziembowskiFaust14*.

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- Theory available to prove the security in (relatively) sound models *DucDziembowskiFaust14*.
- Tools have been developed to automatize the proofs (e.g. *BartheBelaidDupressoirFouqueGrégoireStrub15*)

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- **First Issue:** how to share sensitive data?
- **Related to:**
  - secret sharing *Shamir79*
  - design of error correcting codes with large dual distance *Massey93, CastagnosRennerZémor13*
  - etc.

- **Second Issue:** how to securely process on shared data?
- **Related to:**
  - secure multi-party computation *NikovaRijmenSchläffer2008 ProuffRoche2011*
  - circuit processing in presence of leakage *e.g. GoldwasserRothblum2012*
  - efficient polynomial evaluation *e.g. CarletGoubinProuffQuisquater-Rivain2012, CoronProuffRoche2012, CoronRoyVivek2014*
  - etc.

- (n, d)-SSS: polynomial formulation;
  - generate a random degree-d polynomial

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- Reconstruction with Lagrange's Formula and a subset U of $d+1$:

$$\boxed{Z = \sum_{Z_i \in U} Z_i \times \beta_i \quad ,}$$

where the constants β_i are defined as

$$\beta_i = \prod_{k=1, k\neq i}^{n} \frac{\alpha_k}{\alpha_i + \alpha_k} \quad .$$

Choice of the Public Points α_i

Does the choice of the public points impact the security of SSS in the context of Side-Channel Analysis?

Optimal Number of Shares to Observe

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No influence on the effectiveness of Lagrange's reconstruction BUT the mutual information $(d+1)$-tuple of shares Z_i and Z seems to depend on the α_i *BalashFaustGierlichs15, WangStandaertYu+16*.

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Since the knowledge of $d+1$ shares Z_i is sufficient to recover Z, it is commonly assumed that the optimal number is $d+1$.

Test of template attacks against a $(5, 2)$-SSS $(Z_0, Z_1, \ldots, Z_4)$ of Z

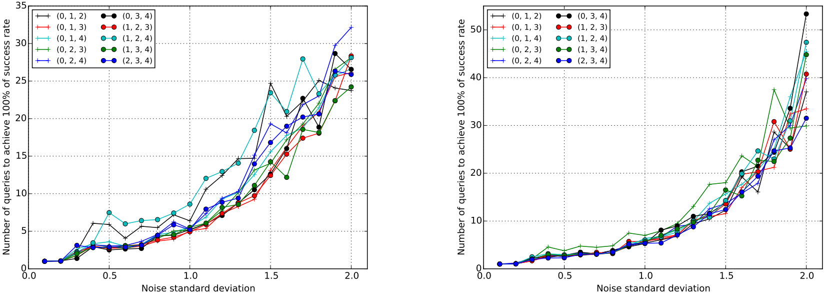

Figure: Number of observations to achieve a success rate of 100%wrt noise standard deviation for two different sets of public points.

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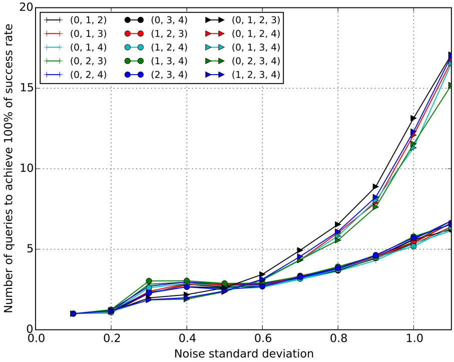

Figure: For different choices of tuples of shares, the number of observations required to achieve a 100% success rate vs the standard deviation of the noise.

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- Rest of this talk: explain this phenomenon.

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- Example *GuruswamiWootters16*:
  - for some $(14, 9)$-SSS sharing
  - Z can be recovered with only 64 bits of information on the Z_i
  - instead of $80 = 10 \times 8$ bits (if 10 shares are targeted)

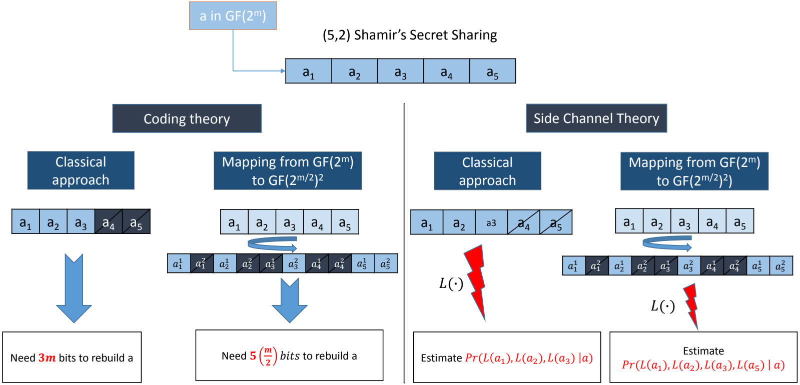

Figure: Side-channel and linear repairing codes for Shamir's sharing.

Z shared into $(Z_1, ..., Z_n)$ s.t. $Z_i = P_Z(\alpha_i)$ and $Z = P_Z(0)$.

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- Necessary Condition: $p_1(0)$, $p_2(0)$, ..., $p_t(0)$ spans vector space of dimension t.

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- Conclusion: more shares are needed (10 instead of 8) but less information is needed (64 bits instead of 80 bits)

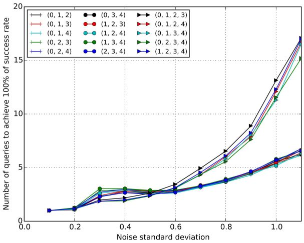
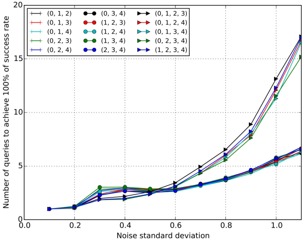


Figure: # of observations to achieve a 100% success rate vs the noise std.



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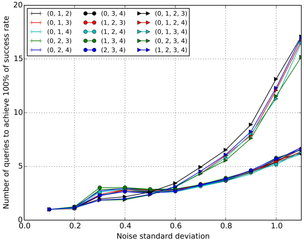

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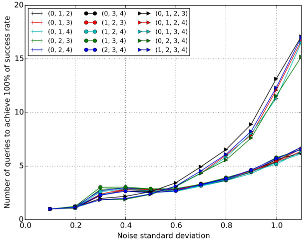

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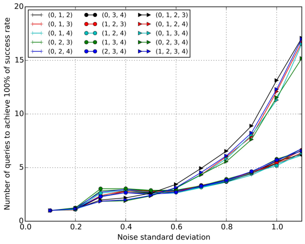

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- In the paper, we combine this property with *GoubinMartinelli11* and *CastagnosRennerZémor13* to improve the efficiency of the secure multiplication over data shared with SSS *Ben-OrGoldwasserWigderson88*.

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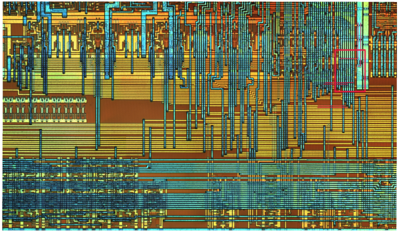
- More works needed to study how to design efficient LERS for given n and d



Thank you for your attention!
Questions/Remarks?